KEY TO ESSENTIALS OF TRIGONOMETRY

KEY

TO

WELLS' ESSENTIALS OF TRIGONOMETRY.

BY

WEBSTER WELLS, S.B.,

ASSOCIATE PROFESSOR OF MATHEMATICS IN THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY.

LEACH, SHEWELL, & SANBORN.
BOSTON AND NEW YORK.

Typography by
J. S. Cushing & Co.

Presswork by
Berwick & Smith.

KEY

TO

WELLS' ESSENTIALS OF TRIGONOMETRY.

CHAPTER I.

Art. 9. — Page 3.

1. $135^\circ = \frac{3\pi}{4}$.

2. $198^\circ = \frac{11\pi}{10}$.

3. $11^\circ 15' = \frac{\pi}{16}$.

4. $37^\circ 30' = \frac{5\pi}{24}$.

5. $29^\circ 15' = \frac{13\pi}{80}$.

6. $174^\circ 22' 30'' = \frac{31\pi}{32}$.

7. $128^\circ 34\frac{2}{7}' = \frac{5\pi}{7}$.

8. $92^\circ 48' 45'' = \frac{33\pi}{64}$.

9. $\frac{1}{2} = \frac{1}{2} \times 57.2957795^\circ \cdots = 28.6478897^\circ \cdots$
$= 28^\circ 38.873382' \cdots = 28^\circ 38' 52.40292'' \cdots$

10. $\frac{3\pi}{5} = \frac{3}{5}$ of $180^\circ = 108^\circ$.

11. $\frac{37\pi}{30} = \frac{37}{30}$ of $180^\circ = 222^\circ$.

12. $\frac{5\pi}{4} = \frac{5}{4}$ of $180^\circ = 225^\circ$.

13. $\frac{3}{4} = \frac{3}{4} \times 57.2957795^\circ \cdots = 42.9718346^\circ \cdots$
$= 42^\circ 58.310076' \cdots = 42^\circ 58' 18.60456'' \cdots$

14. $2 = 2 \times 57.2957795^\circ \cdots = 114.591559^\circ \cdots$
$= 114^\circ 35.49354' \cdots = 114^\circ 35' 29.6124'' \cdots$

15. $\frac{2\pi-1}{3}=\frac{2\pi}{3}-\frac{1}{3}$. $\frac{2\pi}{3}=\frac{2}{3}$ of $180^\circ=120^\circ$.

$$\frac{1}{3}=\frac{1}{3}\times 57.2957795^\circ\cdots=19.0985932^\circ\cdots$$

$$=19^\circ\,5.915592'\cdots \quad =19^\circ\,5'\,54.93552''\cdots$$

$$\therefore \frac{2\pi-1}{3}=120^\circ-19^\circ\,5'\,54.93552''\cdots=100^\circ\,54'\,5.06448''\cdots$$

16. $\frac{\pi-1}{4}=\frac{\pi}{4}-\frac{1}{4}$. $\frac{\pi}{4}=\frac{1}{4}$ of $180^\circ=45^\circ$.

$$\frac{1}{4}=\frac{1}{4}\times 57.2957795^\circ\cdots=14.3239449^\circ\cdots$$

$$=14^\circ\,19.436694'\cdots \quad =14^\circ\,19'\,26.20164''\cdots$$

$$\therefore \frac{\pi-1}{4}=45^\circ-14^\circ\,19'\,26.20164''\cdots=30^\circ\,40'\,33.79836''\cdots$$

CHAPTER II.

Art. 15.— Page 8.

3. Here the opposite side $= 2$, and the adjacent side $= 3$. Therefore the hypotenuse $= \sqrt{2^2 + 3^2} = \sqrt{13}$. Then,

$$\sin A = \frac{2}{\sqrt{13}}, \qquad \cot A = \frac{3}{2}, \qquad \text{vers } A = 1 - \frac{3}{\sqrt{13}},$$

$$\cos A = \frac{3}{\sqrt{13}}, \qquad \sec A = \frac{\sqrt{13}}{3}, \qquad \text{covers } A = 1 - \frac{2}{\sqrt{13}}.$$

$$\csc A = \frac{\sqrt{13}}{2},$$

4. $\text{Sin } A = 1 - \text{covers } A = \frac{2}{5}$.

Here the opposite side $= 2$, and the hypotenuse $= 5$. Therefore the adjacent side $= \sqrt{5^2 - 2^2} = \sqrt{21}$. Then,

$$\cos A = \frac{\sqrt{21}}{5}, \qquad \cot A = \frac{\sqrt{21}}{2}, \qquad \csc A = \frac{5}{2},$$

$$\tan A = \frac{2}{\sqrt{21}}, \qquad \sec A = \frac{5}{\sqrt{21}}, \qquad \text{vers } A = 1 - \frac{\sqrt{21}}{5}.$$

5. Here the hypotenuse $= 4$, and the opposite side $= 1$. Therefore the adjacent side $= \sqrt{4^2 - 1^2} = \sqrt{15}$. Then,

$$\sin A = \frac{1}{4}, \qquad \tan A = \frac{1}{\sqrt{15}}, \qquad \text{vers } A = 1 - \frac{\sqrt{15}}{4},$$

$$\cos A = \frac{\sqrt{15}}{4}, \qquad \cot A = \sqrt{15}, \qquad \text{covers } A = \frac{3}{4}.$$

$$\sec A = \frac{4}{\sqrt{15}},$$

6. $\text{Cos } A = 1 - \text{vers } A = \frac{3}{4}$.

Here the adjacent side $= 3$, and the hypotenuse $= 4$. Therefore the opposite side $= \sqrt{4^2 - 3^2} = \sqrt{7}$. Then,

$$\sin A = \frac{\sqrt{7}}{4}, \qquad \cot A = \frac{3}{\sqrt{7}}, \qquad \csc A = \frac{4}{\sqrt{7}},$$

$$\tan A = \frac{\sqrt{7}}{3}, \qquad \sec A = \frac{4}{3}, \qquad \text{covers } A = 1 - \frac{\sqrt{7}}{4}.$$

7. Here the opposite side $= x$, and the hypotenuse $= y$. Therefore the adjacent side $= \sqrt{y^2 - x^2}$. Then,

$$\sin A = \frac{x}{y}, \quad \cos A = \frac{\sqrt{y^2 - x^2}}{y}, \quad \tan A = \frac{x}{\sqrt{y^2 - x^2}}, \quad \cot A = \frac{\sqrt{y^2 - x^2}}{x}, \quad \sec A = \frac{y}{\sqrt{y^2 - x^2}}, \quad \csc A = \frac{y}{x},$$

$$\text{vers } A = 1 - \frac{\sqrt{y^2 - x^2}}{y}, \quad \text{covers} = 1 - \frac{x}{y}.$$

8. Here the hypotenuse $= 13$, and the adjacent side $= 5$. Therefore the opposite side $= \sqrt{13^2 - 5^2} = 12$. Then,

$$\sin A = \frac{12}{13}, \quad \cos A = \frac{5}{13}, \quad \tan A = \frac{12}{5}, \quad \cot A = \frac{5}{12}, \quad \csc A = \frac{13}{12},$$

$$\text{vers } A = \frac{8}{13}, \quad \text{covers } A = \frac{1}{13}.$$

9. Here the adjacent side $= x$, and the opposite side $= 1$. Therefore the hypotenuse $= \sqrt{x^2 + 1}$. Then,

$$\sin A = \frac{1}{\sqrt{x^2 + 1}}, \quad \cos A = \frac{x}{\sqrt{x^2 + 1}}, \quad \tan A = \frac{1}{x}, \quad \sec A = \frac{\sqrt{x^2 + 1}}{x}, \quad \csc A = \sqrt{x^2 + 1},$$

$$\text{vers } A = 1 - \frac{x}{\sqrt{x^2 + 1}}, \quad \text{covers } A = 1 - \frac{1}{\sqrt{x^2 + 1}}.$$

10. Here the adjacent side $= 8$, and the hypotenuse $= 17$. Therefore the opposite side $= \sqrt{17^2 - 8^2} = 15$. Then,

$$\sin A = \frac{15}{17}, \quad \tan A = \frac{15}{8}, \quad \cot A = \frac{8}{15}, \quad \sec A = \frac{17}{8}, \quad \csc A = \frac{17}{15},$$

$$\text{vers } A = \frac{9}{17}, \quad \text{covers } A = \frac{2}{17}.$$

11. Here the hypotenuse $= \sqrt{a^2 + b^2}$, and the adjacent side $= b$. Therefore the opposite side $= \sqrt{a^2 + b^2 - b^2} = a$. Then,

$$\sin A = \frac{a}{\sqrt{a^2 + b^2}}, \quad \cos A = \frac{b}{\sqrt{a^2 + b^2}}, \quad \tan A = \frac{a}{b}, \quad \cot A = \frac{b}{a}, \quad \csc A = \frac{\sqrt{a^2 + b^2}}{a},$$

$$\text{vers } A = 1 - \frac{b}{\sqrt{a^2 + b^2}}, \quad \text{covers } A = 1 - \frac{a}{\sqrt{a^2 + b^2}}.$$

CHAPTER III.

Art. 53. — Page 30.

3. If A is acute, $450° - A$ is in the first quadrant. Then,

$\sin(450° - A) = \cos A$, $\cos(450° - A) = \sin A$,
$\tan(450° - A) = \cot A$, $\cot(450° - A) = \tan A$,
$\sec(450° - A) = \csc A$, $\csc(450° - A) = \sec A$.

4. If A is acute, $450° + A$ is in the second quadrant. Then,

$\sin(450° + A) = \cos A$, $\cos(450° + A) = -\sin A$,
$\tan(450° + A) = -\cot A$, $\cot(450° + A) = -\tan A$,
$\sec(450° + A) = -\csc A$, $\csc(450° + A) = \sec A$.

5. If A is acute, $540° - A$ is in the second quadrant. Then,

$\sin(540° - A) = \sin A$, $\cos(540° - A) = -\cos A$,
$\tan(540° - A) = -\tan A$, $\cot(540° - A) = -\cot A$,
$\sec(540° - A) = -\sec A$, $\csc(540° - A) = \csc A$.

6. If A is acute, $540° + A$ is in the third quadrant. Then,

$\sin(540° + A) = -\sin A$, $\cos(540° + A) = -\cos A$,
$\tan(540° + A) = \tan A$, $\cot(540° + A) = \cot A$,
$\sec(540° + A) = -\sec A$, $\csc(540° + A) = -\csc A$.

7. If A is acute, $630° - A$ is in the third quadrant. Then,

$\sin(630° - A) = -\cos A$, $\cos(630° - A) = -\sin A$,
$\tan(630° - A) = \cot A$, $\cot(630° - A) = \tan A$,
$\sec(630° - A) = -\csc A$, $\csc(630° - A) = -\sec A$.

8. If A is acute, $900° - A$ is in the second quadrant. Then,

$\sin(900° - A) = \sin A$, $\cos(900° - A) = -\cos A$,
$\tan(900° - A) = -\tan A$, $\cot(900° - A) = -\cot A$,
$\sec(900° - A) = -\sec A$, $\csc(900° - A) = \csc A$.

9. If A is acute, $-90° + A$ is in the fourth quadrant. Then,

$\sin(-90° + A) = -\cos A$, $\cos(-90° + A) = \sin A$,
$\tan(-90° + A) = -\cot A$, $\cot(-90° + A) = -\tan A$,
$\sec(-90° + A) = \csc A$, $\csc(-90° + A) = -\sec A$.

10. If A is acute, $-90^\circ - A$ is in the third quadrant. Then,

$\sin(-90^\circ - A) = -\cos A$, $\cos(-90^\circ - A) = -\sin A$,
$\tan(-90^\circ - A) = \cot A$, $\cot(-90^\circ - A) = \tan A$,
$\sec(-90^\circ - A) = -\csc A$, $\csc(-90^\circ - A) = -\sec A$.

11. If A is acute, $-180^\circ + A$ is in the third quadrant. Then,

$\sin(-180^\circ + A) = -\sin A$, $\cos(-180^\circ + A) = -\cos A$,
$\tan(-180^\circ + A) = \tan A$, $\cot(-180^\circ + A) = \cot A$,
$\sec(-180^\circ + A) = -\sec A$, $\csc(-180^\circ + A) = -\csc A$.

12. If A is acute, $-180^\circ - A$ is in the second quadrant. Then,

$\sin(-180^\circ - A) = \sin A$, $\cos(-180^\circ - A) = -\cos A$,
$\tan(-180^\circ - A) = -\tan A$, $\cot(-180^\circ - A) = -\cot A$,
$\sec(-180^\circ - A) = -\sec A$, $\csc(-180^\circ - A) = \csc A$.

13. If A is acute, $-270^\circ + A$ is in the second quadrant. Then,

$\sin(-270^\circ + A) = \cos A$, $\cos(-270^\circ + A) = -\sin A$,
$\tan(-270^\circ + A) = -\cot A$, $\cot(-270^\circ + A) = -\tan A$,
$\sec(-270^\circ + A) = -\csc A$, $\csc(-270^\circ + A) = \sec A$.

14. If A is acute, $-720^\circ + A$ is in the first quadrant. Then,

$\sin(-720^\circ + A) = \sin A$, $\cos(-720^\circ + A) = \cos A$,
$\tan(-720^\circ + A) = \tan A$, $\cot(-720^\circ + A) = \cot A$,
$\sec(-720^\circ + A) = \sec A$, $\csc(-720^\circ + A) = \csc A$.

Art. 54. — Page 30.

2. $\cos 152^\circ = \cos(180^\circ - 28^\circ) = -\cos 28^\circ$;
or, $= \cos(90^\circ + 62^\circ) = -\sin 62^\circ$.

3. $\tan 522^\circ = \tan(540^\circ - 18^\circ) = -\tan 18^\circ$;
or, $= \tan(450^\circ + 72^\circ) = -\cot 72^\circ$.

4. $\sec(-77^\circ) = \sec(0^\circ - 77^\circ) = \sec 77^\circ$;
or, $= \sec(-90^\circ + 13^\circ) = \csc 13^\circ$.

5. $\csc 230^\circ = \csc(180^\circ + 50^\circ) = -\csc 50^\circ$;
or, $= \csc(270^\circ - 40^\circ) = -\sec 40^\circ$.

6. $\cot(-129^\circ) = \cot(-180^\circ + 51^\circ) = \cot 51^\circ$;
or, $= \cot(-90^\circ - 39^\circ) = \tan 39^\circ$.

7. $\sin 865° = \sin (900° - 35°) = \sin 35°$;
or, $= \sin (810° + 55°) = \cos 55°$.
8. $\cot 83° = \cot (90° - 7°) = \tan 7°$.
9. $\sin (-50°) = \sin (-90° + 40°) = -\cos 40°$.
10. $\sec 165° = \sec (180° - 15°) = -\sec 15°$.
11. $\cos (-303°) = \cos (-270° - 33°) = \sin 33°$.
12. $\tan 520° = \tan (540° - 20°) = -\tan 20°$.
13. $\csc 768° = \csc (810° - 42°) = \sec 42°$.

Table. — Page 31.

Since $120° = 180° - 60°$, we have

$\sin 120° = \sin 60° = \frac{1}{2}\sqrt{3}$. $\cot 120° = -\cot 60° = -\frac{1}{3}\sqrt{3}$.

$\cos 120° = -\cos 60° = -\frac{1}{2}$. $\sec 120° = -\sec 60° = -2$.

$\tan 120° = -\tan 60° = -\sqrt{3}$. $\csc 120° = \csc 60° = \frac{2}{3}\sqrt{3}$.

Since $135° = 180° - 45°$, we have

$\sin 135° = \sin 45° = \frac{1}{2}\sqrt{2}$. $\cot 135° = -\cot 45° = -1$.

$\cos 135° = -\cos 45° = -\frac{1}{2}\sqrt{2}$. $\sec 135° = -\sec 45° = -\sqrt{2}$.

$\tan 135° = -\tan 45° = -1$. $\csc 135° = \csc 45° = \sqrt{2}$.

Since $150° = 180° - 30°$, we have

$\sin 150° = \sin 30° = \frac{1}{2}$. $\cot 150° = -\cot 30° = -\sqrt{3}$.

$\cos 150° = -\cos 30° = -\frac{1}{2}\sqrt{3}$. $\sec 150° = -\sec 30° = -\frac{2}{3}\sqrt{3}$.

$\tan 150° = -\tan 30° = -\frac{1}{3}\sqrt{3}$. $\csc 150° = \csc 30° = 2$.

Since $210° = 180° + 30°$, we have

$\sin 210° = -\sin 30° = -\frac{1}{2}$. $\cot 210° = \cot 30° = \sqrt{3}$.

$\cos 210° = -\cos 30° = -\frac{1}{2}\sqrt{3}$. $\sec 210° = -\sec 30° = -\frac{2}{3}\sqrt{3}$.

$\tan 210° = \tan 30° = \frac{1}{3}\sqrt{3}$. $\csc 210° = -\csc 30° = -2$.

Since $225° = 180° + 45°$, we have

$\sin 225° = -\sin 45° = -\frac{1}{2}\sqrt{2}.$ $\quad \cot 225° = \cot 45° = 1.$

$\cos 225° = -\cos 45° = -\frac{1}{2}\sqrt{2}.$ $\quad \sec 225° = -\sec 45° = -\sqrt{2}.$

$\tan 225° = \tan 45° = 1.$ $\quad \csc 225° = -\csc 45° = -\sqrt{2}.$

Since $240° = 180° + 60°$, we have

$\sin 240° = -\sin 60° = -\frac{1}{2}\sqrt{3}.$ $\quad \cot 240° = \cot 60° = \frac{1}{3}\sqrt{3}.$

$\cos 240° = -\cos 60° = -\frac{1}{2}.$ $\quad \sec 240° = -\sec 60° = -2.$

$\tan 240° = \tan 60° = \sqrt{3}.$ $\quad \csc 240° = -\csc 60° = -\frac{2}{3}\sqrt{3}.$

Since $300° = 360° - 60°$, we have

$\sin 300° = -\sin 60° = -\frac{1}{2}\sqrt{3}.$ $\quad \cot 300° = -\cot 60° = -\frac{1}{3}\sqrt{3}.$

$\cos 300° = \cos 60° = \frac{1}{2}.$ $\quad \sec 300° = \sec 60° = 2.$

$\tan 300° = -\tan 60° = -\sqrt{3}.$ $\quad \csc 300° = -\csc 60° = -\frac{2}{3}\sqrt{3}.$

Since $315° = 360° - 45°$, we have

$\sin 315° = -\sin 45° = -\frac{1}{2}\sqrt{2}.$ $\quad \cot 315° = -\cot 45° = -1.$

$\cos 315° = \cos 45° = \frac{1}{2}\sqrt{2}.$ $\quad \sec 315° = \sec 45° = \sqrt{2}.$

$\tan 315° = -\tan 45° = -1.$ $\quad \csc 315° = -\csc 45° = -\sqrt{2}.$

Since $330° = 360° - 30°$, we have

$\sin 330° = -\sin 30° = -\frac{1}{2}.$ $\quad \cot 330° = -\cot 30° = -\sqrt{3}.$

$\cos 330° = \cos 30° = \frac{1}{2}\sqrt{3}.$ $\quad \sec 330° = \sec 30° = \frac{2}{3}\sqrt{3}.$

$\tan 330° = -\tan 30° = -\frac{1}{3}\sqrt{3}.$ $\quad \csc 330° = -\csc 30° = -2.$

Art. 56. — Page 34.

3. Here the ordinate $= 1$, and the distance $= 4$. Therefore the abscissa $= \pm\sqrt{4^2 - 1^2} = \pm\sqrt{15}$. Then,

$$\cos A = \pm\frac{\sqrt{15}}{4}, \qquad \cot A = \pm\sqrt{15}, \qquad \csc A = 4.$$

$$\tan A = \pm\frac{1}{\sqrt{15}}, \qquad \sec A = \pm\frac{4}{\sqrt{15}},$$

4. Here the abscissa $= 2$ and the ordinate $= 1$, or the abscissa $= -2$ and the ordinate $= -1$. Therefore the distance $= \sqrt{2^2 + 1^2} = \sqrt{5}$. Then,

$$\sin A = \pm\frac{1}{\sqrt{5}}, \qquad \tan A = \frac{1}{2}, \qquad \csc A = \pm\sqrt{5}.$$

$$\cos A = \pm\frac{2}{\sqrt{5}}, \qquad \sec A = \pm\frac{\sqrt{5}}{2},$$

5. Here the distance $= 3$, and the ordinate $= -2$. Therefore the abscissa $= \pm\sqrt{3^2 - 2^2} = \pm\sqrt{5}$. Then,

$$\sin A = -\frac{2}{3}, \qquad \tan A = \mp\frac{2}{\sqrt{5}}, \qquad \sec A = \pm\frac{3}{\sqrt{5}}.$$

$$\cos A = \pm\frac{\sqrt{5}}{3}, \qquad \cot A = \mp\frac{\sqrt{5}}{2},$$

6. Here the ordinate $= 8$ and the abscissa $= -15$, or the ordinate $= -8$ and the abscissa $= 15$. Therefore the distance $= \sqrt{8^2 + 15^2} = 17$. Then,

$$\sin A = \pm\frac{8}{17}, \qquad \cot A = -\frac{15}{8}, \qquad \csc A = \pm\frac{17}{8}.$$

$$\cos A = \mp\frac{15}{17}, \qquad \sec A = \mp\frac{17}{15},$$

7. Here the distance $= 4$, and the abscissa $= 3$. Therefore the ordinate $= \pm\sqrt{4^2 - 3^2} = \pm\sqrt{7}$. Then,

$$\sin A = \pm\frac{\sqrt{7}}{4}, \qquad \tan A = \pm\frac{\sqrt{7}}{3}, \qquad \csc A = \pm\frac{4}{\sqrt{7}}.$$

$$\cos A = \frac{3}{4}, \qquad \cot A = \pm\frac{3}{\sqrt{7}},$$

8. Here the abscissa $= -1$, and the distance $= 2$. Therefore the ordinate $= \pm\sqrt{2^2 - 1^2} = \pm\sqrt{3}$. Then,

$$\sin A = \pm\frac{\sqrt{3}}{2}, \qquad \cot A = \mp\frac{1}{\sqrt{3}}, \qquad \csc A = \pm\frac{2}{\sqrt{3}}.$$

$$\tan A = \mp\sqrt{3}, \qquad \sec A = -2,$$

9. Here the distance $= \sqrt{2}$, and the ordinate $= 1$. Therefore the abscissa $= \pm \sqrt{2-1} = \pm 1$. Then,

$$\sin A = \frac{1}{\sqrt{2}}, \qquad \tan A = \pm 1, \qquad \sec A = \pm \sqrt{2}.$$

$$\cos A = \pm \frac{1}{\sqrt{2}}, \qquad \cot A = \pm 1,$$

10. Here the ordinate $= 2\sqrt{2}$ and the abscissa $= 1$, or the ordinate $= -2\sqrt{2}$ and the abscissa $= -1$. Therefore the distance $= \sqrt{8+1} = 3$. Then,

$$\sin A = \pm \frac{2\sqrt{2}}{3}, \qquad \cot A = \frac{1}{2\sqrt{2}}, \qquad \csc A = \pm \frac{3}{2\sqrt{2}}.$$

$$\cos A = \pm \frac{1}{3}, \qquad \sec A = \pm 3,$$

11. Here the abscissa $= -a$, and the distance $= b$. Therefore the ordinate $= \pm \sqrt{b^2 - a^2}$. Then,

$$\sin A = \pm \frac{\sqrt{b^2-a^2}}{b}, \qquad \cot A = \mp \frac{a}{\sqrt{b^2-a^2}}, \qquad \csc A = \pm \frac{b}{\sqrt{b^2-a^2}}.$$

$$\tan A = \mp \frac{\sqrt{b^2-a^2}}{a}, \qquad \sec A = -\frac{b}{a},$$

12. Here the ordinate $= x$, and the distance $= 1$. Therefore the abscissa $= \pm \sqrt{1-x^2}$. Then,

$$\cos A = \pm \sqrt{1-x^2}, \qquad \cot A = \pm \frac{\sqrt{1-x^2}}{x}, \qquad \csc A = \frac{1}{x}.$$

$$\tan A = \pm \frac{x}{\sqrt{1-x^2}}, \qquad \sec A = \pm \frac{1}{\sqrt{1-x^2}},$$

13. Here the abscissa $= 1$ and the ordinate $= x$, or the abscissa $= -1$ and the ordinate $= -x$. Therefore the distance $= \sqrt{1+x^2}$. Then,

$$\sin A = \pm \frac{x}{\sqrt{1+x^2}}, \qquad \tan A = x, \qquad \csc A = \pm \frac{\sqrt{1+x^2}}{x}.$$

$$\cos A = \pm \frac{1}{\sqrt{1+x^2}}, \qquad \sec A = \pm \sqrt{1+x^2},$$

14. Here the distance $= \sqrt{a^2+b^2}$, and the abscissa $= a$. Therefore the ordinate $= \pm\sqrt{a^2+b^2-a^2} = \pm b$. Then,

$$\sin A = \pm \frac{b}{\sqrt{a^2+b^2}}, \qquad \tan A = \pm \frac{b}{a}, \qquad \csc A = \pm \frac{\sqrt{a^2+b^2}}{b}.$$

$$\cos A = \frac{a}{\sqrt{a^2+b^2}}, \qquad \cot A = \pm \frac{a}{b},$$

CHAPTER IV.

Art. 60. — Page 36.

$$\sin A = \tan A \cos A \text{ (Art. 59)} = \frac{\tan A}{\sec A} = \frac{\tan A}{\sqrt{1+\tan^2 A}} \text{ (Art. 58).}$$

$$\sin A = \frac{1}{\csc A} = \frac{1}{\sqrt{1+\cot^2 A}} \text{ (Art. 58).}$$

$$\sin A = \sqrt{1-\cos^2 A} = \sqrt{1-\frac{1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}.$$

$$\cos A = \frac{1}{\sec A} = \frac{1}{\sqrt{1+\tan^2 A}} \text{ (Art. 58).}$$

$$\cos A = \cot A \sin A \text{ (Art. 59)} = \frac{\cot A}{\csc A} = \frac{\cot A}{\sqrt{1+\cot^2 A}} \text{ (Art. 58).}$$

$$\cos A = \sqrt{1-\sin^2 A} = \sqrt{1-\frac{1}{\csc^2 A}} = \frac{\sqrt{\csc^2 A - 1}}{\csc A}.$$

$$\tan A = \frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1-\sin^2 A}} = \frac{\sqrt{1-\cos^2 A}}{\cos A}.$$

$$\tan A = \frac{1}{\cot A} = \frac{1}{\sqrt{\csc^2 A - 1}} \text{ (Art. 58).}$$

Since the cotangent, secant, and cosecant are the reciprocals of the tangent, cosine, and sine, respectively, we have:

$$\cot A = \frac{\sqrt{1-\sin^2 A}}{\sin A} = \frac{\cos A}{\sqrt{1-\cos^2 A}} = \frac{1}{\sqrt{\sec^2 A - 1}}.$$

$$\sec A = \frac{1}{\sqrt{1-\sin^2 A}} = \frac{\sqrt{1+\cot^2 A}}{\cot A} = \frac{\csc A}{\sqrt{\csc^2 A - 1}}.$$

$$\csc A = \frac{1}{\sqrt{1-\cos^2 A}} = \frac{\sqrt{1+\tan^2 A}}{\tan A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}.$$

CHAPTER V.

Art. 78.—Pages 51 to 53.

3. $$\frac{\sin(x+y)}{\sin(x-y)} = \frac{\sin x\cos y+\cos x\sin y}{\sin x\cos y-\cos x\sin y}$$

$$= \frac{\dfrac{\sin x\cos y}{\cos x\cos y}+\dfrac{\cos x\sin y}{\cos x\cos y}}{\dfrac{\sin x\cos y}{\cos x\cos y}-\dfrac{\cos x\sin y}{\cos x\cos y}} = \frac{\tan x+\tan y}{\tan x-\tan y}.$$

4. $$\frac{\cos(x+y)}{\cos(x-y)} = \frac{\cos x\cos y-\sin x\sin y}{\cos x\cos y+\sin x\sin y}$$

$$= \frac{\dfrac{\cos x\cos y}{\sin x\sin y}-\dfrac{\sin x\sin y}{\sin x\sin y}}{\dfrac{\cos x\cos y}{\sin x\sin y}+\dfrac{\sin x\sin y}{\sin x\sin y}} = \frac{\cot x\cot y-1}{\cot x\cot y+1}.$$

5. $$\frac{\sin(x+y)}{\cos(x-y)} = \frac{\sin x\cos y+\cos x\sin y}{\cos x\cos y+\sin x\sin y}$$

$$= \frac{\dfrac{\sin x\cos y}{\sin x\cos y}+\dfrac{\cos x\sin y}{\sin x\cos y}}{\dfrac{\cos x\cos y}{\sin x\cos y}+\dfrac{\sin x\sin y}{\cin x\cos y}} = \frac{1+\cot x\tan y}{\cot x+\tan y}.$$

6. $$\sin(45^\circ+y) = \sin 45^\circ\cos y+\cos 45^\circ\sin y$$

$$= \frac{1}{\sqrt{2}}\cdot\cos y+\frac{1}{\sqrt{2}}\cdot\sin y \text{ (Art. 16)} = \frac{\sin y+\cos y}{\sqrt{2}}.$$

7. $$\tan(60^\circ-y) = \frac{\tan 60^\circ-\tan y}{1+\tan 60^\circ\tan y} = \frac{\sqrt{3}-\tan y}{1+\sqrt{3}\tan y} \text{ (Art. 17).}$$

8. $$\frac{\sin x+\sin y}{\cos x+\cos y} = \frac{2\sin\frac{1}{2}(x+y)\cos\frac{1}{2}(x-y)}{2\cos\frac{1}{2}(x+y)\cos\frac{1}{2}(x-y)} = \tan\tfrac{1}{2}(x+y).$$

9. $$\frac{\sin x+\sin y}{\cos x-\cos y} = \frac{2\sin\frac{1}{2}(x+y)\cos\frac{1}{2}(x-y)}{-2\sin\frac{1}{2}(x+y)\sin\frac{1}{2}(x-y)} = -\cot\tfrac{1}{2}(x-y).$$

10. By Arts. 74 and 58, $\sin 2x = \dfrac{2\sin x \cos x}{\sin^2 x + \cos^2 x}$.

Dividing each term of the fraction by $\cos^2 x$,

$$\sin 2x = \frac{\dfrac{2\sin x}{\cos x}}{1 + \dfrac{\sin^2 x}{\cos^2 x}} = \frac{2\tan x}{1 + \tan^2 x}.$$

11. $\csc 2x = \dfrac{1}{\sin 2x} = \dfrac{1}{2\sin x \cos x} = \frac{1}{2}\sec x \csc x$ (Art. 57).

12. $\tan x + \cot x = \dfrac{\sin x}{\cos x} + \dfrac{\cos x}{\sin x} = \dfrac{\sin^2 x + \cos^2 x}{\sin x \cos x}$

$$= \frac{1}{\sin x \cos x} = \frac{2}{2\sin x \cos x} = \frac{2}{\sin 2x}.$$

13. $\cot x - \tan x = \cot x - \dfrac{1}{\cot x} = \dfrac{\cot^2 x - 1}{\cot x}$

$$= 2\left(\frac{\cot^2 x - 1}{2\cot x}\right) = 2\cot 2x \text{ (Art. 74).}$$

14. $\dfrac{(1+\tan x)^2 - (1-\tan x)^2}{(1+\tan x)^2 + (1-\tan x)^2}$

$$= \frac{1 + 2\tan x + \tan^2 x - 1 + 2\tan x - \tan^2 x}{1 + 2\tan x + \tan^2 x + 1 - 2\tan x + \tan^2 x}$$

$$= \frac{4\tan x}{2 + 2\tan^2 x} = \frac{2\tan x}{1 + \tan^2 x} = \sin 2x, \text{ by Ex. 10}$$

15. $\sin(x+y)\sin(x-y)$

$= (\sin x \cos y + \cos x \sin y)(\sin x \cos y - \cos x \sin y)$

$= \sin^2 x \cos^2 y - \cos^2 x \sin^2 y$

$= \sin^2 x (1 - \sin^2 y) - (1 - \sin^2 x)\sin^2 y$

$= \sin^2 x - \sin^2 y.$

16. $\cos(x+y)\cos(x-y)$

$= (\cos x \cos y - \sin x \sin y)(\cos x \cos y + \sin x \sin y)$

$= \cos^2 x \cos^2 y - \sin^2 x \sin^2 y$

$= \cos^2 x \cos^2 y - (1 - \cos^2 x)(1 - \cos^2 y)$

$= \cos^2 x \cos^2 y - 1 + \cos^2 x + \cos^2 y - \cos^2 x \cos^2 y$

$= \cos^2 x - (1 - \cos^2 y) = \cos^2 x - \sin^2 y.$

17. $\sec^2 x \csc^2 x = \dfrac{1}{\cos^2 x \sin^2 x} = \dfrac{\sin^2 x + \cos^2 x}{\cos^2 x \sin^2 x}$

$= \dfrac{\sin^2 x}{\cos^2 x \sin^2 x} + \dfrac{\cos^2 x}{\cos^2 x \sin^2 x} = \dfrac{1}{\cos^2 x} + \dfrac{1}{\sin^2 x}$

$= \sec^2 x + \csc^2 x.$

18. $\cos y + \cos (120° + y) + \cos (120° - y)$

$= \cos y + \cos 120° \cos y - \sin 120° \sin y + \cos 120° \cos y + \sin 120° \sin y$

$= \cos y + 2 \cos 120° \cos y = \cos y - \cos y$ (Art. 55) $= 0.$

19. $\sin A \sin (B - C) + \sin B \sin (C - A) + \sin C \sin (A - B)$

$= \sin A (\sin B \cos C - \cos B \sin C) + \sin B (\sin C \cos A - \cos C \sin A)$

$+ \sin C (\sin A \cos B - \cos A \sin B)$

$= \sin A \sin B \cos C - \sin A \cos B \sin C + \sin B \sin C \cos A$

$- \sin B \cos C \sin A + \sin C \sin A \cos B - \sin C \cos A \sin B = 0.$

20. $\cos (A + B) \cos (A - B) + \cos (B + C) \cos (B - C)$

$+ \cos (C + A) \cos (C - A)$

$= \cos^2 A - \sin^2 B + \cos^2 B - \sin^2 C + \cos^2 C - \sin^2 A$, by Ex. 16,

$= \cos 2A + \cos 2B + \cos 2C$ (Art. 74).

21. $\dfrac{\cos x - \cos 3x}{\sin 3x - \sin x} = \dfrac{-(\cos 3x - \cos x)}{\sin 3x - \sin x}$

$= \dfrac{2 \sin \frac{1}{2}(3x + x) \sin \frac{1}{2}(3x - x)}{2 \cos \frac{1}{2}(3x + x) \sin \frac{1}{2}(3x - x)} = \tan 2x.$

22. $\dfrac{\cos 80° + \cos 20°}{\sin 80° - \sin 20°} = \dfrac{2 \cos \frac{1}{2}(80° + 20°) \cos \frac{1}{2}(80° - 20°)}{2 \cos \frac{1}{2}(80° + 20°) \sin \frac{1}{2}(80° - 20°)}$

$= \cot 30° = \sqrt{3}$ (Art. 17).

23. $\sin (x + y + z) = \sin [(x + y) + z]$

$= \sin (x + y) \cos z + \cos (x + y) \sin z$

$= (\sin x \cos y + \cos x \sin y) \cos z + (\cos x \cos y - \sin x \sin y) \sin z$

$= \sin x \cos y \cos z + \cos x \sin y \cos z + \cos x \cos y \sin z - \sin x \sin y \sin z.$

24. $\cos (x + y + z) = \cos [(x + y) + z]$

$= \cos (x + y) \cos z - \sin (x + y) \sin z$

$= (\cos x \cos y - \sin x \sin y) \cos z - (\sin x \cos y + \cos x \sin y) \sin z$

$= \cos x \cos y \cos z - \sin x \sin y \cos z - \sin x \cos y \sin z - \cos x \sin y \sin z.$

25. $\sin 3x = \sin(2x + x) = \sin 2x \cos x + \cos 2x \sin x$
$= 2 \sin x \cos^2 x + (1 - 2 \sin^2 x) \sin x$ (Art. 74)
$= 2 \sin x (1 - \sin^2 x) + \sin x - 2 \sin^3 x$
$= 3 \sin x - 4 \sin^3 x.$

26. $\cos 3x = \cos(2x + x) = \cos 2x \cos x - \sin 2x \sin x$
$= (2 \cos^2 x - 1) \cos x - 2 \sin^2 x \cos x$ (Art. 74)
$= 2 \cos^3 x - \cos x - 2(1 - \cos^2 x) \cos x$
$= 4 \cos^3 x - 3 \cos x.$

27. $\tan 3x = \tan(2x + x) = \dfrac{\tan 2x + \tan x}{1 - \tan 2x \tan x}$

$= \dfrac{\dfrac{2 \tan x}{1 - \tan^2 x} + \tan x}{1 - \dfrac{2 \tan^2 x}{1 - \tan^2 x}}$ (Art. 74)

$= \dfrac{2 \tan x + \tan x - \tan^3 x}{1 - \tan^2 x - 2 \tan^2 x} = \dfrac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x}.$

28. $\sin(2x + y) - 2 \sin x \cos(x + y)$
$= \sin[(x + y) + x] - 2 \sin x \cos(x + y)$
$= \sin(x + y) \cos x + \cos(x + y) \sin x - 2 \sin x \cos(x + y)$
$= \sin(x + y) \cos x - \cos(x + y) \sin x$
$= \sin[(x + y) - x] = \sin y.$

29. $\dfrac{\sin 3x}{\sin x} - \dfrac{\cos 3x}{\cos x} = \dfrac{\sin 3x \cos x - \cos 3x \sin x}{\sin x \cos x}$
$= \dfrac{\sin(3x - x)}{\frac{1}{2} \sin 2x}$ (Art. 74) $= 2.$

30. $1 + \cos 2x \cos 2y = 1 + (2 \cos^2 x - 1)(1 - 2 \sin^2 y)$
$= 2 \cos^2 x + 2 \sin^2 y - 4 \cos^2 x \sin^2 y$
$= 2 \cos^2 x (\sin^2 y + \cos^2 y) + 2 \sin^2 y (\sin^2 x + \cos^2 x) - 4 \cos^2 x \sin^2 y$
$= 2 (\sin^2 x \sin^2 y + \cos^2 x \cos^2 y).$

31. $1 + \tan x \tan 2x = 1 + \dfrac{\sin x \sin 2x}{\cos x \cos 2x}$
$= \dfrac{\cos 2x \cos x + \sin 2x \sin x}{\cos 2x \cos x} = \dfrac{\cos(2x - x)}{\cos 2x \cos x} = \dfrac{1}{\cos 2x} = \sec 2x.$

32. $\sin 4x = 2\sin 2x \cos 2x = 4\sin x \cos x(1 - 2\sin^2 x)$ (Art. 74)
$= 4\sin x \cos x - 8\sin^3 x \cos x.$

33. $\cos 4x = 2\cos^2 2x - 1 = 2(2\cos^2 x - 1)^2 - 1$ (Art. 74)
$= 8\cos^4 x - 8\cos^2 x + 2 - 1 = 8\cos^4 x - 8\cos^2 x + 1.$

34. $\sin 5x = \sin(4x + x) = \sin 4x \cos x + \cos 4x \sin x$
$= 4\sin x \cos^2 x - 8\sin^3 x \cos^2 x + 8\cos^4 x \sin x - 8\cos^2 x \sin x$
$+ \sin x$ (Exs. 32, 33)
$= 4\sin x(1 - \sin^2 x) - 8\sin^3 x(1 - \sin^2 x) + 8(1 - \sin^2 x)^2 \sin x$
$- 8(1 - \sin^2 x)\sin x + \sin x$
$= 4\sin x - 4\sin^3 x - 8\sin^3 x + 8\sin^5 x + 8\sin x - 16\sin^3 x$
$+ 8\sin^5 x - 8\sin x + 8\sin^3 x + \sin x$
$= 5\sin x - 20\sin^3 x + 16\sin^5 x.$

35. $\sin 15° = \sin(45° - 30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$
$= \frac{1}{2}\sqrt{2} \cdot \frac{1}{2}\sqrt{3} - \frac{1}{2}\sqrt{2} \cdot \frac{1}{2} = \frac{1}{4}(\sqrt{6} - \sqrt{2}) = \cos 75°.$
$\cos 15° = \cos(45° - 30°) = \cos 45° \cos 30° + \sin 45° \sin 30°$
$= \frac{1}{2}\sqrt{2} \cdot \frac{1}{2}\sqrt{3} + \frac{1}{2}\sqrt{2} \cdot \frac{1}{2} = \frac{1}{4}(\sqrt{6} + \sqrt{2}) = \sin 75°.$

36. $\tan 15° = \dfrac{1 - \cos 30°}{\sin 30°} = \csc 30° - \cot 30° = 2 - \sqrt{3} = \cot 75°.$
$\cot 15° = \dfrac{1 + \cos 30°}{\sin 30°} = \csc 30° + \cot 30° = 2 + \sqrt{3} = \tan 75°.$

37. $\sin 22° 30' = \sqrt{\dfrac{1 - \cos 45°}{2}} = \sqrt{\dfrac{1 - \frac{1}{2}\sqrt{2}}{2}} = \sqrt{\dfrac{2 - \sqrt{2}}{4}}$
$= \frac{1}{2}\sqrt{2 - \sqrt{2}}.$
$\cos 22° 30' = \sqrt{\dfrac{1 + \cos 45°}{2}} = \sqrt{\dfrac{1 + \frac{1}{2}\sqrt{2}}{2}} = \sqrt{\dfrac{2 + \sqrt{2}}{4}}$
$= \frac{1}{2}\sqrt{2 + \sqrt{2}}.$

38. $\tan 22° 30' = \dfrac{1 - \cos 45°}{\sin 45°} = \csc 45° - \cot 45° = \sqrt{2} - 1.$
$\cot 22° 30' = \dfrac{1 + \cos 45°}{\sin 45°} = \csc 45° + \cot 45° = \sqrt{2} + 1.$

CHAPTER VI.

Art. 91. — Page 57.

2. $\log 6 = \log(2 \times 3) = \log 2 + \log 3 = .3010 + .4771 = .7781.$

3. $\log 14 = \log(2 \times 7) = \log 2 + \log 7 = .3010 + .8451 = 1.1461.$

4. $\log 8 = \log(2 \times 2 \times 2) = \log 2 + \log 2 + \log 2 = 3 \log 2$
$= 3 \times .3010 = .9030.$

5. $\log 12 = \log(2 \times 2 \times 3) = \log 2 + \log 2 + \log 3$
$= 2 \log 2 + \log 3 = .6020 + .4771 = 1.0791.$

6. $\log 15 = \log(3 \times 5) = \log 3 + \log 5 = .4771 + .6990 = 1.1761.$

7. $\log 21 = \log(3 \times 7) = \log 3 + \log 7 = .4771 + .8451 = 1.3222.$

8. $\log 63 = \log(3 \times 3 \times 7) = \log 3 + \log 3 + \log 7 = 2 \log 3 + \log 7$
$= .9542 + .8451 = 1.7993.$

9. $\log 56 = \log(2 \times 2 \times 2 \times 7) = 3 \log 2 + \log 7$
$= .9030 + .8451 = 1.7481.$

10. $\log 84 = \log(2 \times 2 \times 3 \times 7) = 2 \log 2 + \log 3 + \log 7$
$= .6020 + .4771 + .8451 = 1.9242.$

11. $\log 45 = \log(3 \times 3 \times 5) = 2 \log 3 + \log 5 = .9542 + .6990 = 1.6532.$

12. $\log 98 = \log(2 \times 7 \times 7) = \log 2 + 2 \log 7 = .3010 + 1.6902 = 1.9912.$

13. $\log 105 = \log(3 \times 5 \times 7) = \log 3 + \log 5 + \log 7$
$= .4771 + .6990 + .8451 = 2.0212.$

14. $\log 112 = \log(2 \times 2 \times 2 \times 2 \times 7) = 4 \log 2 + \log 7$
$= 1.2040 + .8451 = 2.0491.$

15. $\log 144 = \log(2 \times 2 \times 2 \times 2 \times 3 \times 3) = 4 \log 2 + 2 \log 3$
$= 1.2040 + .9542 = 2.1582.$

16. $\log 216 = \log(2 \times 2 \times 2 \times 3 \times 3 \times 3) = 3 \log 2 + 3 \log 3$
$= .9030 + 1.4313 = 2.3343.$

17. $\log 135 = \log (3 \times 3 \times 3 \times 5) = 3 \log 3 + \log 5$
$= 1.4313 + .6990 = 2.1303.$

18. $\log 168 = \log (2 \times 2 \times 2 \times 3 \times 7) = 3 \log 2 + \log 3 + \log 7$
$= .9030 + .4771 + .8451 = 2.2252.$

19. $\log 147 = \log (3 \times 7 \times 7) = \log 3 + 2 \log 7 = .4771 + 1.6902 = 2.1673.$

20. $\log 375 = \log (3 \times 5 \times 5 \times 5) = \log 3 + 3 \log 5$
$= .4771 + 2.0970 = 2.5741.$

21. $\log 343 = \log (7 \times 7 \times 7) = 3 \log 7 = 2.5353.$

Art. 93. — Page 58.

2. $\log \frac{7}{3} = \log 7 - \log 3 = .8451 - .4771 = .3680.$

3. $\log \frac{10}{7} = \log 10 - \log 7 = 1 - .8451 = .1549.$

4. $\log 3\frac{1}{3} = \log \frac{10}{3} = \log 10 - \log 3 = 1 - .4771 = .5229.$

5. $\log 35 = \log \frac{70}{2} = \log (10 \times 7) - \log 2 = \log 10 + \log 7 - \log 2$
$= 1 + .8451 - .3010 = 1.5441.$

6. $\log \frac{21}{16} = \log 21 - \log 16 = \log (3 \times 7) - \log (2 \times 2 \times 2 \times 2)$
$= \log 3 + \log 7 - 4 \log 2 = .4771 + .8451 - 1.2040 = .1182.$

7. $\log 125 = \log (5 \times 5 \times 5) = 3 \log 5.$
$\log 5 = \log \frac{10}{2} = 1 - \log 2 = 1 - .3010 = .6990.$
$\therefore \log 125 = 3 \times .6990 = 2.0970.$

8. $\log \frac{42}{25} = \log (2 \times 3 \times 7) - \log (5 \times 5) = \log 2 + \log 3 + \log 7 - 2 \log 5.$
$\log 5 = \log \frac{10}{2} = 1 - \log 2 = .6990.$
$\therefore \log \frac{42}{25} = .3010 + .4771 + .8451 - 1.3980 = .2252.$

9. $\log 175 = \log (5 \times 5 \times 7) = 2 \log 5 + \log 7.$
$\log 5 = \log \frac{10}{2} = 1 - \log 2 = .6990.$
$\therefore \log 175 = 1.3980 + .8451 = 2.2431.$

10. $\log 11\frac{1}{9} = \log \frac{100}{9} = \log 100 - \log (3 \times 3)$

$= 2 - 2 \log 3 = 2 - .9542 = 1.0458.$

11. $\log 7\frac{1}{7} = \log \frac{50}{7} = \log \frac{100}{14} = \log 100 - \log (2 \times 7)$

$= 2 - \log 2 - \log 7 = 2 - .3010 - .8451 = .8539.$

12. $\log \frac{35}{6} = \log (5 \times 7) - \log (2 \times 3) = \log 5 + \log 7 - \log 2 - \log 3.$

$\log 5 = \log \frac{10}{2} = 1 - \log 2 = .6990.$

$\therefore \log \frac{35}{6} = .6990 + .8451 - .3010 - .4771 = .7660.$

13. $\log 5\frac{4}{9} = \log \frac{49}{9} = \log (7 \times 7) - \log (3 \times 3) = 2 \log 7 - 2 \log 3$

$= 1.6902 - .9542 = .7360.$

Art. 96. — Page 59.

3. $\log 3^{\frac{3}{5}} = \frac{3}{5} \log 3 = \frac{3}{5} \times .4771 = .2863.$

4. $\log 2^9 = 9 \log 2 = 9 \times .3010 = 2.7090.$

5. $\log 7^5 = 5 \log 7 = 5 \times .8451 = 4.2255.$

6. $\log 5^{\frac{1}{5}} = \frac{1}{5} \log 5.$

$\log 5 = \log \frac{10}{2} = 1 - \log 2 = .6990.$

$\therefore \log 5^{\frac{1}{5}} = \frac{.6990}{5} = .1398.$

7. $\log 12^{\frac{2}{3}} = \frac{2}{3} \log 12.$

$\log 12 = \log (2 \times 2 \times 3) = 2 \log 2 + \log 3 = .6020 + .4771 = 1.0791.$

$\therefore \log 12^{\frac{2}{3}} = \frac{2}{3} \times 1.0791 = .7194.$

8. $\log 21^{\frac{1}{2}} = \frac{1}{2} \log 21.$

$\log 21 = \log (3 \times 7) = \log 3 + \log 7 = .4771 + .8451 = 1.3222.$

$\therefore \log 21^{\frac{1}{2}} = \frac{1.3222}{2} = .6611.$

9. $\log\ 14^4 = 4\log 14.$

$\log\ 14 = \log(2\times 7) = \log 2 + \log 7 = .3010 + .8451 = 1.1461.$

$\therefore \log\ 14^4 = 4\times 1.1461 = 4.5844.$

10. $\log\ 25^{\frac{7}{3}} = \frac{7}{3}\log 25.$

$\log\ 25 = \log\frac{100}{4} = \log 100 - \log(2\times 2)$

$= 2 - 2\log 2 = 2 - .6020 = 1.3980.$

$\therefore \log\ 25^{\frac{7}{3}} = \frac{7}{3}\times 1.3980 = 3.2620.$

11. $\log\ 15^{\frac{5}{6}} = \frac{5}{6}\log 15.$

$\log\ 15 = \log(3\times 5) = \log 3 + \log 5.$

$\log\ 5 = \log\frac{10}{2} = 1 - \log 2 = .6990.$

$\therefore \log\ 15 = .4771 + .6990 = 1.1761.$

$\therefore \log\ 15^{\frac{5}{6}} = \frac{5}{6}\times 1.1761 = .9801.$

12. $\log\ \sqrt{7} = \frac{\log 7}{2} = \frac{.8451}{2} = .4225.$

13. $\log\ \sqrt[3]{3} = \frac{\log 3}{3} = \frac{.4771}{3} = .1590.$

14. $\log\ \sqrt[7]{2} = \frac{\log 2}{7} = \frac{.3010}{7} = .0430.$

15. $\log\ \sqrt[6]{5} = \frac{\log 5}{6}.$

$\log\ 5 = \log\frac{10}{2} = 1 - \log 2 = .6990.$

$\therefore \log\ \sqrt[6]{5} = \frac{.6990}{6} = .1165.$

16. $\log\ \sqrt[4]{35} = \frac{\log 35}{4}.$

$\log\ 35 = \log(5\times 7) = \log 5 + \log 7.$

$\log\ 5 = \log\frac{10}{2} = 1 - \log 2 = .6990.$

$\therefore \log\ 35 = .6990 + .8451 = 1.5441.$

$\therefore \log\ \sqrt[4]{35} = \frac{1.5441}{4} = .3860.$

17. $\log \sqrt[9]{98} = \dfrac{\log 98}{9}$.

$\log 98 = \log (2 \times 7^2) = \log 2 + 2 \log 7 = .3010 + 1.6902 = 1.9912$.

$\therefore \log \sqrt[9]{98} = \dfrac{1.9912}{9} = .2212$.

18. $\log \sqrt[12]{126} = \dfrac{\log 126}{12}$.

$\log 126 = \log (2 \times 3^2 \times 7) = \log 2 + 2 \log 3 + \log 7$
$= .3010 + .9542 + .8451 = 2.1003$.

$\therefore \log \sqrt[12]{126} = \dfrac{2.1003}{12} = .1750$.

20. $\log \left(\dfrac{10}{3}\right)^5 = 5 \log \dfrac{10}{3} = 5 (\log 10 - \log 3) = 5 (1 - .4771)$
$= 5 \times .5229 = 2.6145$.

21. $\log \dfrac{7^{\frac{3}{4}}}{5^{\frac{2}{3}}} = \log 7^{\frac{3}{4}} - \log 5^{\frac{2}{3}} = \dfrac{3}{4} \log 7 - \dfrac{2}{3} \log 5$
$= .6338 - .4660 = .1678$.

22. $\log (3^{\frac{1}{6}} \times 2^{\frac{3}{5}}) = \log 3^{\frac{1}{6}} + \log 2^{\frac{3}{5}} = \dfrac{1}{6} \log 3 + \dfrac{3}{5} \log 2$
$= .0795 + .1806 = .2601$.

23. $\log 3 \sqrt[4]{7} = \log 3 + \log \sqrt[4]{7} = \log 3 + \dfrac{\log 7}{4}$
$= .4771 + .2113 = .6884$.

24. $\log \sqrt{\dfrac{7}{3}} = \dfrac{1}{2} \log \dfrac{7}{3} = \dfrac{\log 7 - \log 3}{2} = \dfrac{.8451 - .4771}{2} = .1840$.

25. $\log \dfrac{\sqrt[3]{7}}{\sqrt[5]{2}} = \log \sqrt[3]{7} - \log \sqrt[5]{2} = \dfrac{\log 7}{3} - \dfrac{\log 2}{5}$
$= .2817 - .0602 = .2215$.

26. $\log \sqrt[3]{\dfrac{28}{5}} = \dfrac{1}{3} \log \dfrac{28}{5} = \dfrac{\log 28 - \log 5}{3}$.

$\log 28 = \log (2^2 \times 7) = 2 \log 2 + \log 7 = .6020 + .8451 = 1.4471$.

$\therefore \log \sqrt[3]{\dfrac{28}{5}} = \dfrac{1.4471 - .6990}{3} = .2494$.

27. $\log \dfrac{\sqrt{42}}{10^{\frac{2}{3}}} = \log \sqrt{42} - \log 10^{\frac{2}{3}} = \dfrac{\log 42}{2} - \dfrac{2}{3} \log 10$.

$\log 42 = \log (2 \times 3 \times 7) = \log 2 + \log 3 + \log 7$
$= .3010 + .4771 + .8451 = 1.6232$.

$\therefore \log \dfrac{\sqrt{42}}{10^{\frac{2}{3}}} = .8116 - .6667 = .1449$.

Art. 98. — Page 60.

2. $\log 18 = \log (2 \times 3^2) = \log 2 + 2 \log 3 = .3010 + .9542 = 1.2552.$
$\therefore \log 1.8 = 0.2552.$

3. $\log 225 = \log (3^2 \times 5^2) = 2 \log 3 + 2 \log 5$
$= .9542 + 1.3980 = 2.3522.$
$\therefore \log 2.25 = 0.3522.$

4. $\log 196 = \log (2^2 \times 7^2) = 2 \log 2 + 2 \log 7$
$= .6020 + 1.6902 = 2.2922.$
$\therefore \log .196 = 9.2922 - 10.$

5. $\log 48 = \log (2^4 \times 3) = 4 \log 2 + \log 3 = 1.2040 + .4771 = 1.6811.$
$\therefore \log .048 = 8.6811 - 10.$

6. $\log 384 = \log (2^7 \times 3) = 7 \log 2 + \log 3 = 2.1070 + .4771 = 2.5841.$
$\therefore \log 38.4 = 1.5841.$

7. $\log 54 = \log (2 \times 3^3) = \log 2 + 3 \log 3 = .3010 + 1.4313 = 1.7323.$
$\therefore \log .0054 = 7.7323 - 10.$

8. $\log 315 = \log (3^2 \times 5 \times 7) = 2 \log 3 + \log 5 + \log 7$
$= .9542 + .6990 + .8451 = 2.4983.$
$\therefore \log .000315 = 6.4983 - 10.$

9. $\log 735 = \log (3 \times 5 \times 7^2) = \log 3 + \log 5 + 2 \log 7$
$= .4771 + .6990 + 1.6902 = 2.8663.$
$\therefore \log 7350 = 3.8663.$

10. $\log 405 = \log (3^4 \times 5) = 4 \log 3 + \log 5$
$= 1.9084 + .6990 = 2.6074.$
$\therefore \log 4.05 = 0.6074.$

11. $\log 448 = \log (2^6 \times 7) = 6 \log 2 + \log 7$
$= 1.8060 + .8451 = 2.6511.$
$\therefore \log .448 = 9.6511 - 10.$

12. $\log 3024 = \log (2^4 \times 3^3 \times 7) = 4 \log 2 + 3 \log 3 + \log 7$
$= 1.2040 + 1.4313 + .8451 = 3.4804.$
$\therefore \log 302.4 = 2.4804.$

13. $\log 6174 = \log (2 \times 3^2 \times 7^3) = \log 2 + 2 \log 3 + 3 \log 7$
$= .3010 + .9542 + 2.5353 = 3.7905.$
$\therefore \log .06174 = 8.7905 - 10.$

14. $\log\ (8.1)^7 = 7 \log 8.1.$
$\log\ \ 81 = \log 3^4 = 4 \log 3 = 1.9084.$
$\therefore \log\ \ 8.1 = 0.9084.$
$\therefore \log\ (8.1)^7 = 7 \times .9084 = 6.3588.$

15. $\log \sqrt[5]{9.6} = \dfrac{\log 9.6}{5}.$
$\log\ \ 96 = \log (2^5 \times 3) = 5 \log 2 + \log 3$
$= 1.5050 + .4771 = 1.9821.$
$\therefore \log\ \ 9.6 = 0.9821.$
$\therefore \log \sqrt[5]{9.6} = \dfrac{0.9821}{5} = .1964.$

16. $\log (22.4)^{\frac{1}{8}} = \dfrac{1}{8} \log 22.4.$
$\log\ \ 224 = \log (2^5 \times 7) = 5 \log 2 + \log 7 = 1.5050 + .8451 = 2.3501.$
$\therefore \log\ \ 22.4 = 1.3501.$
$\therefore \log (22.4)^{\frac{1}{8}} = \dfrac{1.3501}{8} = .1688.$

Art. 105. — Pages 65 and 66.

1. $\log (9.238 \times .9152) = \log 9.238 + \log .9152.$
$\log\ \ 9.238 = 0.9656$
$\log\ \ .9152 = 9.9615 - 10$
$0.9271 \quad = \log 8.454.$

2. $\log (130.36 \times .08237) = \log 130.36 + \log .08237.$
$\log\ 130.36 = 2.1151$
$\log\ .08237 = 8.9157 - 10$
$1.0308 \quad = \log 10.73.$

3. $\log (721.3 \times 3.0528) = \log 721.3 + \log 3.0528.$
$\log\ \ 721.3 = 2.8581$
$\log\ 3.0528 = 0.4847$
$3.3428 = \log 2202.$
Result, $-2202.$

4. $\log (4.3264 \times .050377) = \log 4.3264 + \log .050377.$
$\log\ 4.3264 = 0.6361$
$\log .050377 = 8.7022 - 10$
$9.3383 - 10 = \log .2179.$

5. $\log (.27031 \times .042809) = \log .27031 + \log .042809.$

$\log .27031 = 9.4319 - 10$

$\log .042809 = 8.6315 - 10$

$8.0634 - 10 = \log .01157.$

6. $\log (.063165 \times 11.134) = \log .063165 + \log 11.134.$

$\log .063165 = 8.8005 - 10$

$\log 11.134 = 1.0466$

$9.8471 - 10 = \log .7032.$

Result, $-.7032.$

7. $\log \frac{401.8}{52.37} = \log 401.8 - \log 52.37.$

$\log 401.8 = 2.6040$

$\log 52.37 = 1.7191$

$0.8849 = \log 7.672.$

8. $\log \frac{7.2321}{10.813} = \log 7.2321 - \log 10.813.$

$\log 7.2321 = 0.8592$

$\log 10.813 = 1.0339$

$9.8253 - 10 = \log .6688.$

9. $\log \frac{.3384}{.08659} = \log .3384 - \log .08659.$

$\log .3384 = 9.5294 - 10$

$\log .08659 = 8.9374 - 10$

$0.5920 \quad = \log 3.908.$

Result, $-3.908.$

10. $\log \frac{5.163}{.0051422} = \log 9.163 - \log .0051422.$

$\log 9.163 = 0.9620$

$\log .0051422 = 7.7112 - 10$

$3.2508 \quad = \log 1782.$

11. $\log \frac{22518}{64327} = \log 22518 - \log 64327.$

$\log 22518 = 4.3525$

$\log 64327 = 4.8084$

$9.5441 - 10 = \log .3500.$

12. $\log \frac{.007514}{.015822} = \log .007514 - \log .015822.$

$\log .007514 = 7.8758 - 10$
$\log .015822 = 8.1993 - 10$
$9.6765 - 10 = \log .4748.$
Result, $- .4748.$

13. $\log \frac{3.3681}{12.853 \times .6349}$
$= \log 3.3681 + \operatorname{colog} 12.853 + \operatorname{colog} .6349.$
$\log 3.3681 = 0.5274$
$\operatorname{colog} 12.853 = 8.8910 - 10$
$\operatorname{colog} .6349 = 0.1973$
$9.6157 - 10 = \log .4127.$

14. $\log \frac{15.008 \times .0843}{.06376 \times 4.248}$
$= \log 15.008 + \log .0843 + \operatorname{colog} .06376 + \operatorname{colog} 4.248.$
$\log 15.008 = 1.1763$
$\log .0843 = 8.9258 - 10$
$\operatorname{colog} .06376 = 1.1955$
$\operatorname{colog} 4.248 = 9.3718 - 10$
$0.6694 = \log 4.671.$
Result, $- 4.671.$

15. $\log \frac{2563 \times .03442}{714.8 \times .511}$
$= \log 2563 + \log .03442 + \operatorname{colog} 714.8 + \operatorname{colog} .511.$
$\log 2563 = 3.4087$
$\log .03442 = 8.5368 - 10$
$\operatorname{colog} 714.8 = 7.1458 - 10$
$\operatorname{colog} .511 = 0.2916$
$9.3829 - 10 = \log .2415.$

16. $\log \frac{121.6 \times 9.025}{48.3 \times 3662 \times .0856} = \log 121.6$
$+ \log 9.025 + \operatorname{colog} 48.3 + \operatorname{colog} 3662 + \operatorname{colog} .0856.$
$\log 121.6 = 2.0850$
$\log 9.025 = 0.9554$
$\operatorname{colog} 48.3 = 8.3161 - 10$
$\operatorname{colog} 3662 = 6.4363 - 10$
$\operatorname{colog} .0856 = 1.0675$
$8.8603 - 10 = \log .0725.$
Result, $- .0725.$

17. $\log (23.86)^3 = 3 \times \log 23.86.$

$\log \quad 23.86 = 1.3777$

$\underline{\qquad\qquad 3}$

$4.1331 = \log 13587.$

18. $\log (.532)^8 = 8 \times \log .532.$

$\log \ .532 \quad = 9.7259 - 10$

$\underline{\qquad\qquad\qquad 8}$

$7.8072 - 10 = \log .006415.$

19. $\log (1.0246)^7 = 7 \times \log 1.0246.$

$\log \ 1.0246 \quad = 0.0105$

$\underline{\qquad\qquad 7}$

$0.0735 = \log 1.184.$

Result, $-1.184.$

20. $\log (.09323)^5 = 5 \times \log .09323.$

$\log \ .09323 \quad = 8.9695 - 10$

$\underline{\qquad\qquad\qquad 5}$

$4.8475 - 10 = \log .000007038.$

21. $\log 5^{\frac{2}{3}} = \frac{2}{3} \log 5.$

$\log 5 \ = 0.6990\,; \times \frac{2}{3} = 0.4660$

$= \log 2.924.$

22. $\log (.8)^{\frac{2}{5}} = \frac{2}{5} \log .8.$

$\log \ .8 \quad = 9.9031 - 10$

$\underline{\qquad\qquad\qquad 2}$

$5)\overline{49.8062 - 50}$

$9.9612 - 10 = \log .9146.$

23. $\log (3.16)^{\frac{4}{3}} = \frac{4}{3} \log 3.16.$

$\log \ 3.16 \quad = 0.4997\,; \times \frac{4}{3} = 0.6663$

$= \log 4.638.$

24. $\log (.021)^{\frac{5}{2}} = \frac{5}{2} \log .021.$

$\log \ .021 \quad = 8.3222 - 10$

$\underline{\qquad\qquad\qquad 5}$

$2)\overline{11.6110 - 20}$

$5.8055 - 10 = \log .0000639.$

25. $\log \sqrt{2} = \frac{1}{2} \log 2.$

$\log \quad 2 = 0.3010\,; \div 2 = 0.1505$

$= \log 1.414.$

26. $\log \sqrt[4]{5} = \frac{1}{4} \log 5.$

$\log \quad 5 = 0.6990\,; \div 4 = 0.1747$

$= \log 1.495.$

27. $\log \sqrt[5]{3} = \frac{1}{5} \log 3.$

$\log \quad 3 = 0.4771\,; \div 5 = 0.0954$

$= \log 1.246.$

Result, $-1.246.$

28. $\log \sqrt{.4294} = \frac{1}{2} \log .4294.$

$\log \quad .4294 = 19.6329 - 20\,; \div 2 = 9.8164 - 10$

$= \log .6553.$

29. $\log \sqrt[3]{.02305} = \frac{1}{3} \log .02305.$

$\log \quad .02305 = 28.3626 - 30\,; \div 3 = 9.4542 - 10$

$= \log .2846.$

30. $\log \sqrt[8]{1000} = \frac{1}{8} \log 1000.$

$\log \quad 1000 = 3\,; \div 8 = 0.3750 = \log 2.372.$

31. $\log \sqrt[7]{.00951} = \frac{1}{7} \log .00951.$

$\log \quad .00951 = 67.9782 - 70\,; \div 7 = 9.7112 - 10$

$= \log .5142.$

Result, $-.5142.$

32. $\log \sqrt[5]{.0001011} = \frac{1}{5} \log .0001011.$

$\log \quad .0001011 = 46.0047 - 50\,; \div 5 = 9.2009 - 10$

$= \log .1588.$

35. $\log (2^{\frac{3}{2}} \times 3^{\frac{2}{3}}) = \frac{3}{2} \log 2 + \frac{2}{3} \log 3.$

$\log 2 = .3010\,; \times \frac{3}{2} = .4515$

$\log 3 = .4771\,; \times \frac{2}{3} = .3181$

$.7696 = \log 5.883.$

36. $\log \frac{3^{\frac{5}{8}}}{4^{\frac{2}{3}}} = \frac{5}{8}\log 3 - \frac{2}{3}\log 4.$

$\log\ 3 = .4771;\ \times\frac{5}{8} = .2982$

$\log\ 4 = .6021;\ \times\frac{2}{3} = .4014$

$9.8968 - 10 = \log .7885.$

37. $\log \frac{5^{\frac{3}{7}}}{10^{\frac{2}{9}}} = \frac{3}{7}\log 5 - \frac{2}{9}\log 10.$

$\log\ 5 = .6990;\ \times\frac{3}{7} = .2996$

$\log\ 10 = 1;\ \times\frac{2}{9} = .2222$

$.0774 = \log 1.195.$

38. $\log\left(\frac{6}{7}\right)^{\frac{5}{2}} = \frac{5}{2}(\log 6 - \log 7).$

$\log 6 = .7782$

$\log 7 = .8451$

$9.9331 - 10$

5

$2)19.6655 - 20$

$9.8327 - 10 = \log .6803.$

39. $\log\left(\frac{35}{113}\right)^{\frac{3}{8}} = \frac{3}{8}(\log 35 - \log 113).$

$\log\ 35 = 1.5441$

$\log 113 = 2.0531$

$9.4910 - 10$

3

$8)78.4730 - 80$

$9.8091 - 10 = \log .6443.$

40. $\log\left(\frac{.08726}{.1321}\right)^{\frac{5}{3}} = \frac{5}{3}(\log .08726 - \log .1321).$

$\log .08726 = 8.9408 - 10$

$\log\ .1321 = 9.1209 - 10$

$9.8199 - 10$

5

$3)29.0995 - 30$

$9.6998 - 10 = \log .5010.$

41. $\log \sqrt[8]{\frac{21}{13}} = \frac{1}{8}(\log 21 - \log 13)$.

$\log 21 = 1.3222$
$\log 13 = 1.1139$

$8) \overline{.2083}$
$.0260 = \log 1.062.$

42. $\log \sqrt[9]{\frac{3}{7}} = \frac{1}{9}(\log 3 - \log 7)$.

$\log 3 = .4771$
$\log 7 = .8451$

$9) \overline{89.6320 - 90}$
$9.9591 - 10 = \log .9102.$
Result, $- .9102.$

43. $\log\left(\sqrt[5]{\frac{2}{3}} \div \sqrt[3]{\frac{3}{5}}\right)$
$= \frac{1}{5}(\log 2 - \log 3) - \frac{1}{3}(\log 3 - \log 5).$

$\log 2 = .3010$ $\quad$ $\log 3 = .4771$
$\log 3 = .4771$ $\quad$ $\log 5 = .6990$

$5) \overline{49.8239 - 50}$ $\quad$ $3) \overline{29.7781 - 30}$
$9.9648 - 10$ $\quad$ $9.9260 - 10$
$9.9260 - 10$
$.0388 = \log 1.093.$

44. $\log (\sqrt[3]{2} \times \sqrt[5]{3} \times \sqrt[7]{.01})$
$= \frac{1}{8}\log 2 + \frac{1}{5}\log 3 + \frac{1}{7}\log .01.$

$\log 2 = .3010$; $\div 8 = .0376$
$\log 3 = .4771$; $\div 5 = .0954$
$\log .01 = 68 - 70$; $\div 7 = \overline{9.7143 - 10}$
$9.8473 - 10 = \log .7035.$

45. $\log \sqrt[5]{\frac{3258}{49309}} = \frac{1}{5}(\log 3258 - \log 49309).$

$\log 3258 = 3.5129$
$\log 49309 = 4.6929$

$5) \overline{48.8200 - 50}$
$9.7640 - 10 = \log .5807.$

46. $\log\left(\frac{31.63}{429}\right)^{\frac{3}{17}} = \frac{3}{17}(\log 31.63 - \log 429)$.

$\log 31.63 = 1.5001$

$\log \quad 429 = 2.6325$

$8.8676 - 10$

3

$17)166.6028 - 170$

$9.8002 - 10 = \log .6313.$

Result, $-.6313$.

47. $\log \frac{100^{\frac{2}{3}}}{(.7325)^{\frac{3}{7}}} = \frac{2}{3}\log 100 - \frac{3}{7}\log .7325.$

$\log \quad 100 = 2; \qquad \times\frac{2}{3} = 1.3333$

$\log .7325 = 9.8648 - 10$

3

$69.5944 - 70 \div 7 = 9.9421 - 10$

$1.3912 = \log 24.62.$

48. $\log \frac{\sqrt[3]{.0001289}}{\sqrt[4]{.0008276}} = \frac{1}{3}\log .0001289 - \frac{1}{4}\log .0008276.$

$\log .0001289 = 26.1103 - 30; \div 3 = 8.7034 - 10$

$\log .0008276 = 36.9178 - 40; \div 4 = 9.2294 - 10$

$9.4740 - 10$

$= \log .2979.$

49. $\log \frac{(.7469)^{\frac{5}{3}}}{(.2345)^{\frac{7}{2}}} = \frac{5}{3}\log .7469 - \frac{7}{2}\log .2345.$

$\log .7469 = 9.8732 - 10$

5

$3)29.3660 - 30$

$9.7887 - 10$

$7.7953 - 10$

$1.9934 = \log 98.50.$

$\log .2345 = 9.3701 - 10$

7

$2)15.5907 - 20$

$7.7953 - 10$

50. $\log \frac{\sqrt[11]{.0073}}{(.68291)^{\frac{5}{2}}} = \frac{1}{11}\log .0073 - \frac{5}{2}\log .68291.$

$\log .0073 = 107.8633 - 110$

Dividing by 11, $= 9.8058 - 10$

$9.5857 - 10$

$.2201$

$= \log 1.660.$

$\log .68291 = 9.8343 - 10$

5

$2)19.1715 - 20$

$9.5857 - 10$

51. $\log \dfrac{\sqrt{5.955} \times \sqrt[3]{61.2}}{\sqrt[5]{298.54}}$

$= \frac{1}{2}\log 5.955 + \frac{1}{3}\log 61.2 + \frac{1}{5}\operatorname{colog} 298.54.$

$\log 5.955 = 0.7748 \quad ; \div 2 = 0.3874$
$\log 61.2 = 1.7868 \quad ; \div 3 = 0.5956$
$\operatorname{colog} 298.54 = 47.5250 - 50 \; ; \div 5 = 9.5050 - 10$

$0.4880 = \log 3.076.$

52. $\log (538.2 \times .0005969)^{\frac{1}{8}} = \frac{1}{8}(\log 538.2 + \log .0005969).$

$\log 538.2 = 2.7310$
$\log .0005969 = 6.7759 - 10$

$8)79.5069 - 80$
$9.9384 - 10 = \log .8678.$

53. $\log [(18.9503)^{11} \times (.1)^{14}]$
$= 11 \times \log 18.9503 + 14 \times \log .1.$
$\log 18.9503 = 1.2777 \; ; \times 11 = 14.0547$
$\log .1 = 9 - 10 \; ; \times 14 = 6. - 20$

$.0547 = \log 1.134.$

54. $\log \sqrt[6]{3734.9 \times .00001108} = \frac{1}{6}(\log 3734.9 + \log .00001108).$

$\log 3734.9 = 3.5723$
$\log .00001108 = 5.0445 - 10$

$6)58.6168 - 60$
$9.7695 - 10 = \log .5881.$

55. $\log [(2.6317)^{\frac{3}{4}} \times (.71272)^{\frac{2}{5}}]$
$= \frac{3}{4}\log 2.6317 + \frac{2}{5}\log .71272.$

$\log 2.6317 = .4203$
3
$4)1.2609$
$.3152$
$9.9412 - 10$

$.2564 = \log 1.805.$

$\log .71272 = 9.8529 - 10$
2
$5)49.7058 - 50$
$9.9412 - 10$

56. $\log \dfrac{\sqrt[3]{.008193} \times (.06285)^{\frac{3}{2}}}{.98342}$

$= \frac{1}{3}\log .008193 + \frac{3}{2}\log .06285 + \operatorname{colog} .98342.$

$\log .008193 = 27.9134 - 30\,;\ \div 3 = 9.3045 - 10$
$\log\ .06285 = 8.7983 - 10$
$\times\ 3$
$16.3949 - 20\,;\ \div 2 = 8.1974 - 10$
$\operatorname{colog} .98342 = 0.0072$
$7.5091\bar{1} - 10$
$= \log .003229.$

57. $\log (\sqrt{.035} \times \sqrt[6]{.62667} \times \sqrt[3]{.0072103})$

$= \frac{1}{2}\log .035 + \frac{1}{6}\log .62667 + \frac{1}{3}\log .0072103.$

$\log\ .035 = 18.5441 - 20\,;\ \div 2 = 9.2720 - 10$
$\log\ .62667 = 59.7971 - 60\,;\ \div 6 = 9.9662 - 10$
$\log .0072103 = 27.8579 - 30\,;\ \div 3 = 9.2860 - 10$
$8.5242 - 10$
$= \log .03344.$

CHAPTER VII.

Art. 110. — Pages 69 to 72.

1.

$a = c \sin A.$
$\log c = 1.0492$
$\log \sin A = 9.8378$
$\log a = 0.8870$
$\therefore a = 7.708.$

$b = c \cos A.$
$\log c = 1.0492$
$\log \cos A = 9.8606$
$\log b = 0.9098$
$\therefore b = 8.124.$

2.

$b = a \tan B.$
$\log a = 2.8629$
$\log \tan B = 0.4121$
$\log b = 3.2750$
$\therefore b = 1883.$

$c = \frac{a}{\cos B}.$
$\log a = 2.8629$
$\log \cos B = 9.5576$
$\log c = 3.3053$
$\therefore c = 2019.5.$

3.

$a = \frac{b}{\tan B}.$
$\log b = 1.6785$
$\log \tan B = 0.2916$
$\log a = 1.3869$
$\therefore a = 24.37.$

$c = \frac{b}{\sin B}.$
$\log b = 1.6785$
$\log \sin B = 9.9496$
$\log c = 1.7289$
$\therefore c = 53.56.$

4.

$\sin A = \frac{a}{c}.$
$\log a = 9.7952$
$\log c = 9.9590$
$\log \sin A = 9.8362$
$\therefore A = 43^\circ\ 17.9'.$

$b = \frac{a}{\tan A}.$
$\log a = 9.7952$
$\log \tan A = 9.9742$
$\log b = 9.8210$
$\therefore b = .6622.$

5.

$\tan A = \frac{a}{b}.$
$\log a = 0.6990$
$\log b = 0.3010$
$\log \tan A = 0.3980$
$\therefore A = 68^\circ\ 12.2'.$

$c = \frac{a}{\sin A}.$
$\log a = 0.6990$
$\log \sin A = 9.9678$
$\log c = 0.7312$
$\therefore c = 5.385.$

6. $b = \frac{a}{\tan A}$.

$\log a = 1.9212$
$\log \tan A = 0.4912$
$\log b = 1.4300$
$\therefore b = 26.91.$

$c = \frac{a}{\sin A}$.

$\log a = 1.9212$
$\log \sin A = 9.9785$
$\log c = 1.9427$
$\therefore c = 87.64.$

7. $a = c \cos B$

$\log c = 8.4359$
$\log \cos B = 9.9276$
$\log a = 8.3635$
$\therefore a = .02309.$

$b = c \sin B.$

$\log c = 8.4359$
$\log \sin B = 9.7262$
$\log b = 8.1621$
$\therefore b = .01452.$

8. $\cos A = \frac{b}{c}$.

$\log b = 0.4604$
$\log c = 0.7084$
$\log \cos A = 9.7520$
$\therefore A = 55° 36.1'.$

$a = b \tan A.$

$\log b = 0.4604$
$\log \tan A = 0.1645$
$\log a = 0.6249$
$\therefore a = 4.216.$

9. $a = b \tan A.$

$\log b = 3.6281$
$\log \tan A = 0.1179$
$\log a = 3.7460$
$\therefore a = 5571.$

$c = \frac{b}{\cos A}$.

$\log b = 3.6281$
$\log \cos A = 9.7826$
$\log c = 3.8455$
$\therefore c = 7007.$

10. $\tan A = \frac{a}{b}$.

$\log a = 2.0043$
$\log b = 2.0645$
$\log \tan A = 9.9398$
$\therefore A = 41° 2.4'.$

$c = \frac{a}{\sin A}$.

$\log a = 2.0043$
$\log \sin A = 9.8173$
$\log c = 2.1870$
$\therefore c = 153.8.$

11. $b = \frac{a}{\tan A}$.

$\log a = 2.1995$
$\log \tan A = 9.9752$
$\log b = 2.2243$
$\therefore b = 167.6.$

$c = \frac{a}{\sin A}$.

$\log a = 2.1995$
$\log \sin A = 9.8368$
$\log c = 2.3627$
$\therefore c = 230.5.$

12. $a = c \sin A$. $b = c \cos A$.

$\log c = 1.5531$ $\log c = 1.5531$

$\log \sin A = 9.9314$ $\log \cos A = 9.7162$

$\log a = 1.4845$ $\log b = 1.2693$

$\therefore a = 30.51$. $\therefore b = 18.59$.

13. $\sin A = \frac{a}{c}$. $b = \frac{a}{\tan A}$.

$\log a = 2.3100$ $\log a = 2.3100$

$\log c = 2.4398$ $\log \tan A = 0.0436$

$\log \sin A = 9.8702$ $\log b = 2.2664$

$\therefore A = 47^\circ 52.5'$. $\therefore b = 184.7$.

14. $a = \frac{b}{\tan B}$. $c = \frac{b}{\sin B}$.

$\log b = 0.2158$ $\log b = 0.2158$

$\log \tan B = 9.7614$ $\log \sin B = 9.6990$

$\log a = 0.4544$ $\log c = 0.5168$

$\therefore a = 2.847$. $\therefore c = 3.287$.

15. $a = b \tan A$. $c = \frac{b}{\cos A}$.

$\log b = 1.1220$ $\log b = 1.1220$

$\log \tan A = 9.6114$ $\log \cos A = 9.9665$

$\log a = 0.7334$ $\log c = 1.1555$

$\therefore a = 5.4125$. $\therefore c = 14.306$.

16. $a = c \cos B$. $b = c \sin B$.

$\log c = 9.8611$ $\log c = 9.8611$

$\log \cos B = 9.9922$ $\log \sin B = 9.2747$

$\log a = 9.8533$ $\log b = 9.1358$

$\therefore a = .7133$. $\therefore b = .1367$.

17. $\tan A = \frac{a}{b}$. $c = \frac{a}{\sin A}$.

$\log a = 2.8051$ $\log a = 2.8051$

$\log b = 2.7000$ $\log \sin A = 9.8957$

$\log \tan A = 0.1051$ $\log c = 2.9094$

$\therefore A = 51^\circ 51.9'$. $\therefore c = 811.7$.

18. $b = \frac{a}{\tan A}$.

$\log a = 2.3092$
$\log \tan A = 0.6832$
$\log b = 1.6260$
$\therefore b = 42.27.$

$c = \frac{a}{\sin A}$.

$\log a = 2.3092$
$\log \sin A = 9.9908$
$\log c = 2.3184$
$\therefore c = 208.15.$

19. $\cos A = \frac{b}{c}$.

$\log b = 8.3974$
$\log c = 8.6805$
$\log \cos A = 9.7169$
$\therefore A = 58° 35.7'.$

$a = b \tan A.$

$\log b = 8.3974$
$\log \tan A = 0.2143$
$\log a = 8.6117$
$\therefore a = .0409$

20. $b = a \tan B.$

$\log a = 3.2731$
$\log \tan B = 8.6085$
$\log b = 1.8816$
$\therefore b = 76.13.$

$c = \frac{a}{\cos B}$.

$\log a = 3.2731$
$\log \cos B = 9.9996$
$\log c = 3.2735$
$\therefore c = 1877.$

21. $\tan A = \frac{a}{b}$.

$\log a = 1.3922$
$\log b = 1.5188$
$\log \tan A = 9.8734$
$\therefore A = 36° 45.9'.$

$c = \frac{a}{\sin A}$.

$\log a = 1.3922$
$\log \sin A = 9.7771$
$\log c = 1.6151$
$\therefore c = 41.22.$

22. $\cos A = \frac{b}{c}$.

$\log b = 0.1574$
$\log c = 0.5397$
$\log \cos A = 9.6177$
$\therefore A = 65° 30'.$

$a = b \tan A.$

$\log b = 0.1574$
$\log \tan A = 0.3413$
$\log a = 0.4987$
$\therefore a = 3.153.$

23. $a = c \cos B.$

$\log c = 4.5706$
$\log \cos B = 9.9975$
$\log a = 4.5681$
$\therefore a = 36992.$

$b = c \sin B.$

$\log c = 4.5706$
$\log \sin B = 9.0337$
$\log b = 3.6043$
$\therefore b = 4021.$

24. $a = b \tan A.$

$\log b = 2.3011$
$\log \tan A = 0.3122$
$\log a = 2.6133$
$\therefore a = 410.5.$

$c = \frac{b}{\cos A}.$

$\log b = 2.3011$
$\log \cos A = 9.6415$
$\log c = 2.6596$
$\therefore c = 456.7.$

25. $\tan A = \frac{a}{b}.$

$\log a = 2.5316$
$\log b = 2.3649$
$\log \tan A = 0.1667$
$\therefore A = 55° 44.1'.$

$c = \frac{a}{\sin A}.$

$\log a = 2.5316$
$\log \sin A = 9.9172$
$\log c = 2.6144$
$\therefore c = 411.5.$

26. $\sin A = \frac{a}{c}.$

$\log a = 0.2327$
$\log c = 0.3012$
$\log \sin A = 9.9315$
$\therefore A = 58° 40'.$

$b = \frac{a}{\tan A}.$

$\log a = 0.2327$
$\log \tan A = 0.2155$
$\log b = 0.0172$
$\therefore b = 1.0405.$

27. $b = a \tan B.$

$\log a = 9.9144$
$\log \tan B = 9.5968$
$\log b = 9.5112$
$\therefore b = .3245.$

$c = \frac{a}{\cos B}.$

$\log a = 9.9144$
$\log \cos B = 9.9685$
$\log c = 9.9459$
$\therefore c = .8828.$

28. $a = c \sin A.$

$\log c = 2.4403$
$\log \sin A = 9.9828$
$\log a = 2.4231$
$\therefore a = 264.9.$

$b = c \cos A$

$\log c = 2.4403$
$\log \cos A = 9.4402$
$\log b = 1.8805$
$\therefore b = 75.95.$

29. $a = \frac{b}{\tan B}.$

$\log b = 2.0800$
$\log \tan B = 9.8329$
$\log a = 2.2471$
$\therefore a = 176.64.$

$c = \frac{b}{\sin B}.$

$\log b = 2.0800$
$\log \sin B = 9.7503$
$\log c = 2.3297$
$\therefore c = 213.65.$

30. $\tan A = \frac{a}{b}$.

$\log a = 1.0046$
$\log b = 1.2381$
$\log \tan A = 9.7665$
$\therefore A = 30° 17.2'$.

$c = \frac{a}{\sin A}$.

$\log a = 1.0046$
$\log \sin A = 9.7027$
$\log c = 1.3019$
$\therefore c = 20.04$.

31. $c = 2b \cos A$.

$\log 2b = 1.8462$
$\log \cos A = 9.5553$
$\log c = 1.4015$
$\therefore c = 25.206$.

34. $\cos A = \frac{\frac{1}{2}c}{a}$.

$\log \frac{1}{2}c = 1.7268$
$\log a = 1.8989$
$\log \cos A = 9.8279$
$\therefore A = 47° 42.9'$.

32. $a = \frac{\frac{1}{2}c}{\cos B}$.

$\log \frac{1}{2}c = 0.1886$
$\log \cos B = 9.9493$
$\log a = 0.2393$
$\therefore a = 1.735$.

35. $a = \frac{\frac{1}{2}c}{\cos A}$.

$\log \frac{1}{2}c = 9.4489$
$\log \cos A = 9.5274$
$\log a = 9.9215$
$\therefore a = .8346$.

33. $c = 2b \sin \frac{1}{2} C$.

$\log 2b = 3.6239$
$\log \sin \frac{1}{2} C = 9.8116$
$\log c = 3.4355$
$\therefore c = 2725.6$.

36. $a = \frac{\frac{1}{2}c}{\sin \frac{1}{2} C}$.

$\log \frac{1}{2}c = 1.6788$
$\log \sin \frac{1}{2} C = 9.9864$
$\log a = 1.6924$
$\therefore a = 49.25$.

37. Let O be the centre, and AB any side of the pentagon. Join OA, and draw OC perpendicular to AB.

Then $AB = 2\ OA \sin AOC = 24 \sin 36°$.

$\log 24 = 1.3802$
$\log \sin 36° = 9.7692$
$\log AB = 1.1494$
$\therefore AB = 14.106$.

38. Let A be the point of observation, B the top of the tower, and C its base.

Then $BC = AC \tan A = 100 \tan 38°$.

$\log 100 = 2.0000$
$\log \tan 38° = 9.8928$
$\log BC = 1.8928$
$\therefore BC = 78.12$.

39. Let B be the top and C the base of the tower, and A the extremity of its shadow.

Then $$\tan A = \frac{BC}{AC} = \frac{103.7}{167.3}.$$

$$\log 103.7 = 2.0157$$
$$\log 167.3 = 2.2235$$
$$\log \tan A = 9.7922$$
$$\therefore A = 31^\circ\, 47.1'.$$

40. Let AB be the chord, and O the centre of the circle.
Join OA and OB, and draw OC perpendicular to AB.

Then $$\sin AOC = \frac{AC}{OA} = \frac{513.5}{1634}.$$

$$\log 513.5 = 2.7105$$
$$\log 1634 = 3.2132$$
$$\log \sin AOC = 9.4973$$
$$\therefore AOC = 18^\circ\, 18.95'.$$
$$\therefore AOB = 36^\circ\, 37.9'.$$

41. Let A be the top of the mountain, B the remotest point visible, and O the centre of the earth.

Then in the right triangle OAB, $OA = 3956 + 1\frac{1}{4} = 3957.25$, and $OB = 3956$.

Hence $$AB = \sqrt{\overline{OA}^2 - \overline{OB}^2}$$
$$= \sqrt{(OA + OB)(OA - OB)}$$
$$= \sqrt{7913.25 \times 1.25}.$$

$$\log 7913.25 = 3.8984$$
$$\log 1.25 = 0.0969$$
$$2)\overline{3.9953}$$
$$\log AB = 1.9976$$
$$\therefore AB = 99.45.$$

42. Let AB and BC be consecutive sides of the pentagon.
Join AC, and draw BD perpendicular to AC.

Then $$AC = 2\, AB \cos BAC = 14.056 \cos 36^\circ.$$

$$\log 14.056 = 1.1478$$
$$\log \cos 36^\circ = 9.9080$$
$$\log AC = 1.0558$$
$$\therefore AC = 11.371.$$

43. Let A denote the angle of elevation.

Then $$\tan A = \frac{238}{660}.$$

$$\log 238 = 2.3766$$
$$\log 660 = 2.8195$$
$$\log \tan A = 9.5571$$
$$\therefore\ A = 19^\circ 50'.$$

44. Let A be the position of the buoy, B the top of the light-house, and C its base.

Then $$AC = BC \cot BAC = 133 \cot 18^\circ 25'.$$

$$\log 133 = 2.1239$$
$$\log \cot 18^\circ 25' = 0.4776$$
$$\log AC = 2.6015$$
$$\therefore\ AC = 399.5.$$

45. Let H be the position of the headland, S the first point of observation, and S' the second.

Then in the right triangle HSS', $SS' = 16.38$, and $\angle SHS' = 33^\circ$.

Hence $$HS = 16.38 \cot 33^\circ,$$

and $$HS' = \frac{16.38}{\sin 33^\circ}.$$

$$\log 16.38 = 1.2143$$
$$\log \cot 33^\circ = 0.1875$$
$$\log HS = 1.4018$$
$$\therefore\ HS = 25.22.$$

$$\log 16.38 = 1.2143$$
$$\log \sin 33^\circ = 9.7361$$
$$\log HS' = 1.4782$$
$$\therefore\ HS' = 30.07.$$

46. Let AB be the chord, and O the centre of the circle.

Join OA, and draw OC perpendicular to AB.

Then $$OA = \frac{AC}{\sin AOC} = \frac{20.68}{\sin 72^\circ 48.5'}.$$

$$\log 20.68 = 1.3156$$
$$\log \sin 72^\circ 48.5' = 9.9801$$
$$\log OA = 1.3355$$
$$\therefore\ OA = 21.65.$$

47. Let O be the centre, and AB any side of the octagon. Join OA, and draw OC perpendicular to AB.

Then $$OC = AC \cot AOC = 6 \cot 22^\circ 30',$$

and $$OA = \frac{AC}{\sin AOC} = \frac{6}{\sin 22^\circ 30'}.$$

$$\log 6 = 0.7782$$
$$\log \cot 22^\circ 30' = 0.3828$$
$$\log OC = 1.1610$$
$$\therefore\ OC = 14.487.$$

$$\log 6 = 0.7782$$
$$\log \sin 22^\circ 30' = 9.5828$$
$$\log OA = 1.1954$$
$$\therefore\ OA = 15.682.$$

48. Let A be the position of the observer, B the top of the pole, and C its foot.

Then $$AC = BC \cot BAC = 80 \cot 10^\circ.$$

$$\log 80 = 1.9031$$
$$\log \cot 10^\circ = 0.7537$$
$$\log AC = 2.6568$$
$$\therefore\ AC = 453.7.$$

49. Let O be the centre, and AB any diagonal of the pentagon. Join OA, and draw OC perpendicular to AB.

Then $$OA = \frac{AC}{\sin AOC} = \frac{16.415}{\sin 72^\circ}.$$

$$\log 16.415 = 1.2152$$
$$\log \sin 72^\circ = 9.9782$$
$$\log OA = 1.2370$$
$$\therefore\ OA = 17.26.$$

50. Let B be the top, and C the foot of the tower, and A the extremity of the base line.

Then $$BC = AC \tan BAC = 1000 \tan 21^\circ 16' 37''.$$

$$\log 1000 = 3.0000$$
$$\log \tan 21^\circ 16' 37'' = 9.5904$$
$$\log BC = 2.5904$$
$$\therefore\ BC = 389.4.$$

51. Let AB be the chord, and O the centre of the circle.
Join OA, and draw OC perpendicular to AB.

Then $AB = 2\,OA \sin AOC = 1446.58 \sin 17° 36.5'$.

$$\log 1446.58 = 3.1604$$
$$\log \sin 17° 36.5' = 9.4807$$
$$\log AB = 2.6411$$
$$\therefore\ AB = 437.6.$$

52. Let O be the centre, and AB any side of the hexagon.
Join OA, and draw OC perpendicular to AB.

Then $AB = 2\,OC \tan AOC = 10 \tan 30°$.

$$\log 10 = 1.0000$$
$$\log \tan 30° = 9.7614$$
$$\log AB = 0.7614$$
$$\therefore\ AB = 5.773.$$

53. Let A be the position of the first boat, and B of the second; let C be the top of the light-house, and D its foot.

Then $AD = CD \cot CAD = 200 \cot 14°$,

and $BD = CD \cot CBD = 200 \cot 32°$.

$$\log 200 = 2.3010$$
$$\log \cot 14° = 0.6032$$
$$\log AD = 2.9042$$
$$\therefore\ AD = 802.$$

$$\log 200 = 2.3010$$
$$\log \cot 32° = 0.2042$$
$$\log BD = 2.5052$$
$$\therefore\ BD = 320.1.$$
$$\therefore\ AB = AD - BD = 481.9.$$

54. Let A be the position of the light-house, and B, C, and D, the positions of the ship at 7 A.M., 7.30 A.M., and 10 A.M., respectively.

Then $BC = AB \tan BAC = 10.32 \tan 18° 13'$.

$$\log 10.32 = 1.0136$$
$$\log \tan 18° 13' = 9.5174$$
$$\log BC = 0.5310$$
$$\therefore\ BC = 3.396.$$

Therefore the rate of the ship is 2×3.396, or 6.792 miles an hour.

Again, $$\tan BAD = \frac{BD}{AB} = \frac{20.376}{10.32}.$$

$$\log 20.376 = 1.3091$$
$$\log 10.32 = \underline{1.0136}$$
$$\log \tan BAD = 0.2955$$
$$\therefore\ BAD = 63^\circ\ 8.4'.$$

Therefore the bearing of the light-house at 10 A.M. is $63^\circ\ 8.4'$ west of north.

Art. 112. — Page 74.

2. $2K = a^2 \cot A.$

$$2 \log a = 2.6916$$
$$\log \cot A = \underline{0.4485}$$
$$\log 2K = 3.1401$$
$$2K = 1380.6$$
$$\therefore\ K = 690.3.$$

3. $2K = a^2 \tan B.$

$$2 \log a = 9.8290$$
$$\log \tan B = \underline{9.6510}$$
$$\log 2K = 9.4800$$
$$2K = .302$$
$$\therefore\ K = .151.$$

4. $2K = ab.$

$$\log a = 2.1741$$
$$\log b = \underline{1.8824}$$
$$\log 2K = 4.0565$$
$$2K = 11389$$
$$\therefore\ K = 5695.$$

5. $2K = b\sqrt{(c+b)(c-b)}.$

$$\log b = 9.4851$$
$$\tfrac{1}{2}\log (c+b) = 9.9924$$
$$\tfrac{1}{2}\log (c-b) = \underline{9.7748}$$
$$\log 2K = 9.2523$$
$$2K = .17876$$
$$\therefore\ K = .08938.$$

6. $4K = c^2 \sin 2A.$

$$2 \log c = 4.5708$$
$$\log \sin 2A = \underline{9.9455}$$
$$\log 4K = 4.5163$$
$$4K = 32831$$
$$\therefore\ K = 8208.$$

7. $2K = b^2 \tan A.$

$$2 \log b = 7.4332$$
$$\log \tan A = \underline{0.2190}$$
$$\log 2K = 7.6522$$
$$2K = .00449$$
$$\therefore\ K = .002245.$$

8. $2K = a\sqrt{(c+a)(c-a)}.$

$$\log a = 9.9694$$
$$\tfrac{1}{2}\log (c+a) = 0.2851$$
$$\tfrac{1}{2}\log (c-a) = \underline{0.1341}$$
$$\log 2K = 0.3886$$
$$2K = 2.447$$
$$\therefore\ K = 1.223.$$

9. $4K = c^2 \sin 2B.$

$$2 \log c = 2.8718$$
$$\log \sin 2B = \underline{9.7604}$$
$$\log 4K = 2.6322$$
$$4K = 428.7$$
$$\therefore\ K = 107.2.$$

10. $2K = b^2 \cot B.$

$$2 \log b = 9.0574$$
$$\log \cot B = \underline{0.2508}$$
$$\log 2K = 9.3082$$
$$2K = .2033$$
$$\therefore\ K = .1017.$$

CHAPTER IX.

Art. 121. — Page 84.

2. $C = 180° - 115° 10' = 64° 50'$.

$b = a \sin B \csc A.$

$\log a = 1.0000$
$\log \sin B = 9.9890$
$\log \csc A = 0.2107$
$\log b = 1.1997$
$\therefore b = 15.837.$

$c = a \sin C \csc A.$

$\log a = 1.0000$
$\log \sin C = 9.9567$
$\log \csc A = 0.2107$
$\log c = 1.1674$
$\therefore c = 14.703.$

3. $A = 180° - 154° = 26°$.

$a = b \sin A \csc B.$

$\log b = 9.9051$
$\log \sin A = 9.6418$
$\log \csc B = 0.1015$
$\log a = 9.6484$
$\therefore a = .445.$

$c = b \sin C \csc B.$

$\log b = 9.9051$
$\log \sin C = 9.9909$
$\log \csc B = 0.1015$
$\log c = 9.9975$
$\therefore c = .9942.$

4. $C = 180° - 80° 35' = 99° 25'$.

$a = c \sin A \csc C.$

$\log c = 8.5051$
$\log \sin A = 9.7706$
$\log \csc C = 0.0059$
$\log a = 8.2816$
$\therefore a = .01913.$

$b = c \sin B \csc C.$

$\log c = 8.5051$
$\log \sin B = 9.8453$
$\log \csc C = 0.0059$
$\log b = 8.3563$
$\therefore b = .02272.$

5. $B = 180° - 120° 55' = 59° 5'$.

$a = b \sin A \csc B.$

$\log b = 1.4625$
$\log \sin A = 9.9996$
$\log \csc B = 0.0666$
$\log a = 1.5287$
$\therefore a = 33.78.$

$c = b \sin C \csc B.$

$\log b = 1.4625$
$\log \sin C = 9.7390$
$\log \csc B = 0.0666$
$\log c = 1.2681$
$\therefore c = 18.54.$

6. $A = 180° - 139° 23' = 40° 37'$.

$b = a \sin B \csc A$.
$\log a = 0.7340$
$\log \sin B = 9.9954$
$\log \csc A = 0.1865$
$\log b = 0.9159$
$\therefore b = 8.24$.

$c = a \sin C \csc A$.
$\log a = 0.7340$
$\log \sin C = 9.8170$
$\log \csc A = 0.1865$
$\log c = 0.7375$
$\therefore c = 5.464$.

7. $B = 180° - 158° 54' = 21° 6'$.

$a = c \sin A \csc C$.
$\log c = 8.2068$
$\log \sin A = 9.7613$
$\log \csc C = 0.0796$
$\log a = 8.0477$
$\therefore a = .011162$.

$b = c \sin B \csc C$.
$\log c = 8.2068$
$\log \sin B = 9.5563$
$\log \csc C = 0.0796$
$\log b = 7.8427$
$\therefore b = .006962$.

8. $B = 180° - 114° 28' = 65° 32'$.

$b = a \sin B \csc A$.
$\log a = 2.6021$
$\log \sin B = 9.9591$
$\log \csc A = 0.0895$
$\log b = 2.6507$
$\therefore b = 447.4$.

$c = a \sin C \csc A$.
$\log a = 2.6021$
$\log \sin C = 9.9375$
$\log \csc A = 0.0895$
$\log c = 2.6291$
$\therefore c = 425.7$.

9. $C = 180° - 125° 13' = 54° 47'$.

$a = b \sin A \csc B$.
$\log b = 2.4973$
$\log \sin A = 9.9652$
$\log \csc B = 0.0723$
$\log a = 2.5348$
$\therefore a = 342.6$.

$c = b \sin C \csc B$.
$\log b = 2.4973$
$\log \sin C = 9.9122$
$\log \csc B = 0.0723$
$\log c = 2.4818$
$\therefore c = 303.3$.

10. $A = 180° - 75° 28' 18'' = 104° 31' 42''$.

$a = c \sin A \csc C$.
$\log c = 0.8954$
$\log \sin A = 9.9858$
$\log \csc C = 0.1628$
$\log a = 1.0440$
$\therefore a = 11.067$.

$b = c \sin B \csc C$.
$\log c = 0.8954$
$\log \sin B = 9.7248$
$\log \csc C = 0.1628$
$\log b = 0.7830$
$\therefore b = 6.067$.

Art. 122. — Page 85.

2. $\tan \frac{1}{2}(A-C) = \frac{a-c}{a+c} \tan \frac{1}{2}(A+C)$. $\quad b = a \sin B \csc A$.

$a - c = 12$	$\log = 1.0792$	$\log a = 1.4314$
$a + c = 42$	$\operatorname{colog} = 8.3768$	$\log \sin B = 9.8569$
$\frac{1}{2}(A+C) = 67°$	$\log \tan = 0.3721$	$\log \csc A = 0.0080$
	$\log \tan \frac{1}{2}(A-C) = 9.8281$	$\log b = 1.2963$
	$\therefore \frac{1}{2}(A-C) = 33° 56.7'$.	$\therefore b = 19.78$.

$\therefore A = \frac{1}{2}(A+C) + \frac{1}{2}(A-C) = 100° 56.7'$,
and $C = \frac{1}{2}(A+C) - \frac{1}{2}(A-C) = 33° 3.3'$.

3. $\tan \frac{1}{2}(A-B) = \frac{a-b}{a+b} \tan \frac{1}{2}(A+B)$. $\quad c = a \sin C \csc A$.

$a - b = 139$	$\log = 2.1430$	$\log a = 2.6866$
$a + b = 833$	$\operatorname{colog} = 7.0794$	$\log \sin C = 9.8941$
$\frac{1}{2}(A+B) = 64° 12'$	$\log \tan = 0.3157$	$\log \csc A = 0.0030$
	$\log \tan \frac{1}{2}(A-B) = 9.5381$	$\log c = 2.5837$
	$\therefore \frac{1}{2}(A-B) = 19° 2.7'$.	$\therefore c = 383.5$.

$\therefore A = \frac{1}{2}(A+B) + \frac{1}{2}(A-B) = 83° 14.7'$,
and $B = \frac{1}{2}(A+B) - \frac{1}{2}(A-B) = 45° 9.3'$.

4. $\tan \frac{1}{2}(C-B) = \frac{c-b}{c+b} \tan \frac{1}{2}(C+B)$. $\quad a = b \sin A \csc B$.

$c - b = 1.265$	$\log = 0.1021$	$\log b = 0.3621$
$c + b = 5.869$	$\operatorname{colog} = 9.2315$	$\log \sin A = 9.9459$
$\frac{1}{2}(C+B) = 59°$	$\log \tan = 0.2212$	$\log \csc B = 0.1987$
	$\log \tan \frac{1}{2}(C-B) = 9.5548$	$\log a = 0.5067$
	$\therefore \frac{1}{2}(C-B) = 19° 44.2'$.	$\therefore a = 3.211$.

$\therefore C = \frac{1}{2}(C+B) + \frac{1}{2}(C-B) = 78° 44.2'$,
and $B = \frac{1}{2}(C+B) - \frac{1}{2}(C-B) = 39° 15.8'$.

5. $\tan \frac{1}{2}(B-A) = \frac{b-a}{b+a} \tan \frac{1}{2}(B+A)$. $\quad c = a \sin C \csc A$.

$b - a = .063$	$\log = 8.7993$	$\log a = 9.4771$
$b + a = .663$	$\operatorname{colog} = 0.1785$	$\log \sin C = 9.9137$
$\frac{1}{2}(B+A) = 27° 32'$	$\log \tan = 9.7171$	$\log \csc A = 0.3790$
	$\log \tan \frac{1}{2}(B-A) = 8.6949$	$\log c = 9.7698$
	$\therefore \frac{1}{2}(B-A) = 2° 50.2'$.	$\therefore c = .5886$.

$\therefore B = \frac{1}{2}(B+A) + \frac{1}{2}(B-A) = 30° 22.2'$,
and $A = \frac{1}{2}(B+A) - \frac{1}{2}(B-A) = 24° 41.8'$.

6. $\tan \frac{1}{2}(B-C) = \frac{b-c}{b+c} \tan \frac{1}{2}(B+C)$. $a = b \sin A \csc B$.

$b-c = 835.8$	$\log = 2.9221$	$\log b = 3.0763$
$b+c = 1548.4$	$\operatorname{colog} = 6.8101$	$\log \sin A = 9.6460$
$\frac{1}{2}(B+C) = 76° 52'$	$\log \tan = 0.6320$	$\log \csc B = 0.2254$
	$\log \tan \frac{1}{2}(B-C) = 0.3642$	$\log a = 2.9477$
	$\therefore \frac{1}{2}(B-C) = 66° 37.1'$.	$\therefore a = 886.6$.

$\therefore B = \frac{1}{2}(B+C) + \frac{1}{2}(B-C) = 143° 29.1'$,
and $C = \frac{1}{2}(B+C) - \frac{1}{2}(B-C) = 10° 14.9'$.

7. $\tan \frac{1}{2}(C-A) = \frac{c-a}{c+a} \tan \frac{1}{2}(C+A)$. $b = a \sin B \csc A$.

$c-a = 4.039$	$\log = 0.6063$	$\log a = 0.8692$
$c+a = 18.839$	$\operatorname{colog} = 8.7249$	$\log \sin B = 9.9962$
$\frac{1}{2}(C+A) = 48° 47'$	$\log \tan = 0.0575$	$\log \csc A = 0.2411$
	$\log \tan \frac{1}{2}(C-A) = 9.3887$	$\log b = 1.1065$
	$\therefore \frac{1}{2}(C-A) = 13° 45.1'$.	$\therefore b = 12.78$.

$\therefore C = \frac{1}{2}(C+A) + \frac{1}{2}(C-A) = 62° 32.1'$,
and $A = \frac{1}{2}(C+A) - \frac{1}{2}(C-A) = 35° 1.9'$.

8. $\tan \frac{1}{2}(A-B) = \frac{a-b}{a+b} \tan \frac{1}{2}(A+B)$. $c = a \sin C \csc A$.

$a-b = 11.66$	$\log = 1.0667$	$\log a = 1.7265$
$a+b = 94.88$	$\operatorname{colog} = 8.0228$	$\log \sin C = 9.9913$
$\frac{1}{2}(A+B) = 50° 43.5'$	$\log \tan = 0.0874$	$\log \csc A = 0.0657$
	$\log \tan \frac{1}{2}(A-B) = 9.1769$	$\log c = 1.7835$
	$\therefore \frac{1}{2}(A-B) = 8° 32.8'$.	$\therefore c = 60.74$.

$\therefore A = \frac{1}{2}(A+B) + \frac{1}{2}(A-B) = 59° 16.3'$,
and $B = \frac{1}{2}(A+B) - \frac{1}{2}(A-B) = 42° 10.7'$.

9. $\tan \frac{1}{2}(C-B) = \frac{c-b}{c+b} \tan \frac{1}{2}(C+B)$. $a = b \sin A \csc B$.

$c-b = .02424$	$\log = 8.3845$	$\log b = 8.4262$
$c+b = .0776$	$\operatorname{colog} = 1.1101$	$\log \sin A = 9.9545$
$\frac{1}{2}(C+B) = 32° 6.5'$	$\log \tan = 9.7976$	$\log \csc B = 0.4453$
	$\log \tan \frac{1}{2}(C-B) = 9.2922$	$\log a = 8.8260$
	$\therefore \frac{1}{2}(C-B) = 11° 5.3'$.	$\therefore a = .06699$.

$\therefore C = \frac{1}{2}(C+B) + \frac{1}{2}(C-B) = 43° 11.8'$,
and $B = \frac{1}{2}(C+B) - \frac{1}{2}(C-B) = 21° 1.2'$.

10. $\tan \frac{1}{2}(C-A) = \frac{c-a}{c+a} \tan \frac{1}{2}(C+A)$. $\qquad b = a \sin B \csc A$.

$c - a = 16.56$	$\log = 1.2191$	$\log a = 1.7108$
$c + a = 119.32$	$\operatorname{colog} = 7.9233$	$\log \sin B = 9.9923$
$\frac{1}{2}(C+A) = 50^\circ 23' 43''$	$\log \tan = 0.0823$	$\log \csc A = 0.1842$
	$\log \tan \frac{1}{2}(C-A) = 9.2247$	$\log b = 1.8873$
	$\therefore \frac{1}{2}(C-A) = 9^\circ 31.4' = 9^\circ 31' 24''$.	$\therefore b = 77.14$.

$\therefore C = \frac{1}{2}(C+A) + \frac{1}{2}(C-A) = 59^\circ 55' 7''$,
and $A = \frac{1}{2}(C+A) - \frac{1}{2}(C-A) = 40^\circ 52' 19''$.

Art. 123. — Page 88.

3. Here $s = 4.5$, $s - a = 2.5$, $s - b = 1.5$, $s - c = .5$.

$\log (s-a) = 0.3979$	$\log r = 9.8099$
$\log (s-b) = 0.1761$	$\log (s-b) = 0.1761$
$\log (s-c) = 9.6990$	$\log \tan \frac{1}{2} B = 9.6338$
$\operatorname{colog} s = 9.3468$	$\frac{1}{2} B = 23^\circ 17.1'$.
$2) 9.6198$	$\therefore B = 46^\circ 34.2'$.
$\log r = 9.8099$	$\log r = 9.8099$
$\log r = 9.8099$	$\log (s-c) = 9.6990$
$\log (s-a) = 0.3979$	$\log \tan \frac{1}{2} C = 0.1109$
$\log \tan \frac{1}{2} A = 9.4120$	$\frac{1}{2} C = 52^\circ 14.2'$.
$\frac{1}{2} A = 14^\circ 28.7'$.	$\therefore C = 104^\circ 28.4'$.
$\therefore A = 28^\circ 57.4'$.	

Check, $A + B + C = 180^\circ$.

4. Here $s = 8.5$, $s - a = 4.5$, $s - b = 1.5$, $s - c = 2.5$.

$\log (s-a) = 0.6532$	$\log r = 0.1489$
$\log (s-b) = 0.1761$	$\log (s-b) = 0.1761$
$\log (s-c) = 0.3979$	$\log \tan \frac{1}{2} B = 9.9728$
$\operatorname{colog} s = 9.0706$	$\frac{1}{2} B = 43^\circ 12.4'$.
$2) 0.2978$	$\therefore B = 86^\circ 24.8'$.
$\log r = 0.1489$	$\log r = 0.1489$
$\log (s-a) = 0.6532$	$\log (s-c) = 0.3979$
$\log \tan \frac{1}{2} A = 9.4957$	$\log \tan \frac{1}{2} C = 9.7510$
$\frac{1}{2} A = 17^\circ 23.2'$.	$\frac{1}{2} C = 29^\circ 24.5'$.
$\therefore A = 34^\circ 46.4'$.	$\therefore C = 58^\circ 49'$.

Check, $A + B + C = 180^\circ 0.2$.

5. Here $s = 7.4$, $s - a = 1.8$, $s - b = 3.1$, $s - c = 2.5$.

$\log (s-a) = 0.2553$
$\log (s-b) = 0.4914$
$\log (s-c) = 0.3979$
$\operatorname{colog} s = 9.1308$
$2)\underline{0.2754}$
$\log r = 0.1377$
$\log (s-a) = 0.2553$
$\log \tan \frac{1}{2}A = 9.8824$
$\frac{1}{2}A = 37^\circ 20'.$
$\therefore A = 74^\circ 40'.$

$\log r = 0.1377$
$\log (s-b) = 0.4914$
$\log \tan \frac{1}{2}B = 9.6463$
$\frac{1}{2}B = 23^\circ 53.2'.$
$\therefore B = 47^\circ 46.4'.$

$\log r = 0.1377$
$\log (s-c) = 0.3979$
$\log \tan \frac{1}{2}C = 9.7398$
$\frac{1}{2}C = 28^\circ 46.7'.$
$\therefore C = 57^\circ 33.4'.$

Check, $A + B + C = 179^\circ 59.8'$.

6. Here $s = .344$, $s - a = .114$, $s - b = .084$, $s - c = .146$.

$\log (s-a) = 9.0569$
$\log (s-b) = 8.9243$
$\log (s-c) = 9.1644$
$\operatorname{colog} s = 0.4634$
$2)\underline{7.6090}$
$\log r = 8.8045$
$\log (s-a) = 9.0569$
$\log \tan \frac{1}{2}A = 9.7476$
$\frac{1}{2}A = 29^\circ 13'.$
$\therefore A = 58^\circ 26'.$

$\log r = 8.8045$
$\log (s-b) = 8.9243$
$\log \tan \frac{1}{2}B = 9.8802$
$\frac{1}{2}B = 37^\circ 11.9'.$
$\therefore B = 74^\circ 23.8'.$

$\log r = 8.8045$
$\log (s-c) = 9.1644$
$\log \tan \frac{1}{2}C = 9.6401$
$\frac{1}{2}C = 23^\circ 35.3'.$
$\therefore C = 47^\circ 10.6'.$

Check, $A + B + C = 180^\circ 0.4'$.

7. Here $s = 120.2$, $s - a = 40.9$, $s - b = 26$, $s - c = 53.3$.

$\log (s-a) = 1.6117$
$\log (s-b) = 1.4150$
$\log (s-c) = 1.7267$
$\operatorname{colog} s = 7.9201$
$2)\underline{2.6735}$
$\log r = 1.3367$
$\log (s-a) = 1.6117$
$\log \tan \frac{1}{2}A = 9.7250$
$\frac{1}{2}A = 27^\circ 57.7'.$
$\therefore A = 55^\circ 55.4'.$

$\log r = 1.3367$
$\log (s-b) = 1.4150$
$\log \tan \frac{1}{2}B = 9.9217$
$\frac{1}{2}B = 39^\circ 51.9'.$
$\therefore B = 79^\circ 43.8'.$

$\log r = 1.3367$
$\log (s-c) = 1.7267$
$\log \tan \frac{1}{2}C = 9.6100$
$\frac{1}{2}C = 22^\circ 10'.$
$\therefore C = 44^\circ 20'.$

Check, $A + B + C = 179^\circ 59.2'$.

8. Here $s = 542$, $s - a = 221$, $s - b = 181$, $s - c = 140$.

$\log (s-a) = 2.3444$
$\log (s-b) = 2.2577$
$\log (s-c) = 2.1461$
$\operatorname{colog} s = 7.2660$
$2\overline{)4.0142}$
$\log r = 2.0071$
$\log (s-a) = 2.3444$
$\log \tan \frac{1}{2} A = 9.6627$
$\frac{1}{2} A = 24^\circ 42.1'$.
$\therefore A = 49^\circ 24.2'$.

$\log r = 2.0071$
$\log (s-b) = 2.2577$
$\log \tan \frac{1}{2} B = 9.7494$
$\frac{1}{2} B = 29^\circ 19'$.
$\therefore B = 58^\circ 38'$.

$\log r = 2.0071$
$\log (s-c) = 2.1461$
$\log \tan \frac{1}{2} C = 9.8610$
$\frac{1}{2} C = 35^\circ 58.9'$.
$\therefore C = 71^\circ 57.8'$.

Check, $A + B + C = 180^\circ$.

9. Here $s = .936$, $s - a = .295$, $s - b = .407$, $s - c = .234$.

$\log (s-a) = 9.4698$
$\log (s-b) = 9.6096$
$\log (s-c) = 9.3692$
$\operatorname{colog} s = 0.0287$
$2\overline{)8.4773}$
$\log r = 9.2386$
$\log (s-a) = 9.4698$
$\log \tan \frac{1}{2} A = 9.7688$
$\frac{1}{2} A = 30^\circ 25.4'$.
$\therefore A = 60^\circ 50.8'$.

$\log r = 9.2386$
$\log (s-b) = 9.6096$
$\log \tan \frac{1}{2} B = 9.6290$
$\frac{1}{2} B = 23^\circ 3.1'$.
$\therefore B = 46^\circ 6.2'$.

$\log r = 9.2386$
$\log (s-c) = 9.3692$
$\log \tan \frac{1}{2} C = 9.8694$
$\frac{1}{2} C = 36^\circ 30.8'$.
$\therefore C = 73^\circ 1.6'$.

Check, $A + B + C = 179^\circ 58.6'$.

10. Here $s = 6.989$, $s - a = 3.97$, $s - b = .258$, $s - c = 2.761$.

$\log (s-a) = 0.5988$
$\log (s-b) = 9.4116$
$\log (s-c) = 0.4411$
$\operatorname{colog} s = 9.1556$
$2\overline{)9.6071}$
$\log r = 9.8035$
$\log (s-a) = 0.5988$
$\log \tan \frac{1}{2} A = 9.2047$
$\frac{1}{2} A = 9^\circ 6.2'$.
$\therefore A = 18^\circ 12.4'$.

$\log r = 9.8035$
$\log (s-b) = 9.4116$
$\log \tan \frac{1}{2} B = 0.3919$
$\frac{1}{2} B = 67^\circ 55.3'$.
$\therefore B = 135^\circ 50.6'$.

$\log r = 9.8035$
$\log (s-c) = 0.4411$
$\log \tan \frac{1}{2} C = 9.3624$
$\frac{1}{2} C = 12^\circ 58.3'$.
$\therefore C = 25^\circ 56.6'$.

Check, $A + B + C = 179^\circ 59.6'$.

Art. 127. — Pages 91 and 92.

6. Since b is $< a$, there is but one solution, corresponding to the acute value of B.

$$\sin B = \frac{b \sin A}{a}. \qquad c = a \sin C \csc A.$$

$\log b = 0.5551$	$\log a = 0.7059$
$\operatorname{colog} a = 9.2941$	$\log \sin C = 9.9884$
$\log \sin A = 9.9530$	$\log \csc A = 0.0470$
$\log \sin B = 9.8022$	$\log c = 0.7413$
$\therefore B = 39° 21.3'$,	$\therefore c = 5.511.$

and $C = 180° - 103° 11.3' = 76° 48.7'.$

7. Since b is $> c$, and C is acute, there will be two solutions, one solution, or no solution, according as $\log \sin B$ is negative, zero, or positive.

$$\sin B = \frac{b \sin C}{c}. \qquad a_1 = b \sin A_1 \csc B. \qquad a_2 = b \sin A_2 \csc B.$$

$\log b = 1.8739$	$\log b = 1.8739$	$\log b = 1.8739$
$\operatorname{colog} c = 8.2062$	$\log \sin A_1 = 9.9408$	$\log \sin A_2 = 9.0316$
$\log \sin C = 9.6615$	$\log \csc B = 0.2584$	$\log \csc B = 0.2584$
$\log \sin B = 9.7416$	$\log a_1 = 2.0731$	$\log a_2 = 1.1639$
$\therefore B_1 = 33° 28.4'$,	$\therefore a_1 = 118.33.$	$\therefore a_2 = 14.58.$

and $B_2 = 146° 31.6'.$

$\therefore A_1 = 180° - 60° 46.4' = 119° 13.6',$

and $A_2 = 180° - 173° 49.6' = 6° 10.4'.$

8. Since c is $< b$, there is but one solution, corresponding to the acute value of C.

$$\sin C = \frac{c \sin B}{b}. \qquad a = b \sin A \csc B.$$

$\log c = 9.2971$	$\log b = 9.3687$
$\operatorname{colog} b = 0.6313$	$\log \sin A = 9.4825$
$\log \sin B = 9.9757$	$\log \csc B = 0.0243$
$\log \sin C = 9.9041$	$\log a = 8.8755$
$\therefore C = 53° 18.9'$,	$\therefore a = .07508.$

and $A = 180° - 162° 18.9' = 17° 41.1'.$

9. Since a is $< c$, there is but one solution, corresponding to the acute value of A.

$$\sin A = \frac{a \sin C}{c}. \qquad b = a \sin B \csc A.$$

$\log a = 0.0294$	$\log a = 0.0294$
$\operatorname{colog} c = 9.7670$	$\log \sin B = 9.8916$
$\log \sin C = 9.7228$	$\log \csc A = 0.4808$
$\log \sin A = 9.5192$	$\log b = 0.4018$
$\therefore A = 19° 18.1'$,	$\therefore b = 2.522.$

and $B = 180° - 51° 11.1' = 128° 48.9'$.

10. $\sin A = \dfrac{a \sin B}{b}$. $\quad c_1 = a \sin C_1 \csc A.$ $\quad c_2 = a \sin C_2 \csc A.$

$\log a = 9.2704$	$\log a = 9.2704$	$\log a = 9.2704$
$\operatorname{colog} b = 0.7696$	$\log \sin C_1 = 9.7795$	$\log \sin C_2 = 9.4314$
$\log \sin B = 9.9524$	$\log \csc A = 0.0076$	$\log \csc A = 0.0076$
$\log \sin A = 9.9924$	$\log c_1 = 9.0575$	$\log c_2 = 8.7094$
$\therefore A_1 = 79° 20'$,	$\therefore c_1 = .11416.$	$\therefore c_2 = .05121$

and $A_2 = 100° 40'$.

$\therefore C_1 = 180° - 143° \quad = 37°$,

and $C_2 = 180° - 164° 20' = 15° 40'$.

11. Since c is $> a$, and A is obtuse, the triangle is impossible.

12. Since b is $< c$, there is but one solution, corresponding to the acute value of B.

$$\sin B = \frac{b \sin C}{c}. \qquad a = b \sin A \csc B.$$

$\log b = 1.7016$	$\log b = 1.7016$
$\operatorname{colog} c = 8.1752$	$\log \sin A = 9.9232$
$\log \sin C = 9.7340$	$\log \csc B = 0.3892$
$\log \sin B = 9.6108$	$\log a = 2.0140$
$\therefore B = 24° 5.4'$,	$\therefore \quad = 103.3.$

and $A = 180° - 56° 54.4' = 123° 5.6'$.

13. $\sin C = \dfrac{c \sin A}{a}$. $\qquad b = a \tan B.$

$\log c = 1.0000$	$\log a = 0.9373$
$\operatorname{colog} a = 9.0627$	$\log \tan B = 9.7623$
$\log \sin A = 9.9373$	$\log b = 0.6996$
0.0000	$\therefore b = 5.007.$

$\therefore C = 90°$,

and $B = 90° - 59°57' = 30° 3'$.

14. $\sin C = \dfrac{c \sin B}{b}$. $\quad a_1 = b \sin A_1 \csc B$. $\quad a_2 = b \sin A_2 \csc B$.

$\log c = 0.8351$	$\log b = 0.7127$	$\log b = 0.7127$
$\operatorname{colog} b = 9.2873$	$\log \sin A_1 = 9.9695$	$\log \sin A_2 = 9.5939$
$\log \sin B = 9.8422$	$\log \csc B = 0.1578$	$\log \csc B = 0.1578$
$\log \sin C = 9.9646$	$\log a_1 = 0.8400$	$\log a_2 = 0.4644$
$\therefore C_1 = 67^\circ 10'$,	$\therefore a_1 = 6.918$.	$\therefore a_2 = 2.913$.

and $C_2 = 112^\circ 50'$.

$\therefore A_1 = 180^\circ - 111^\circ 13' = 68^\circ 47'$,

and $A_2 = 180^\circ - 156^\circ 53' = 23^\circ 7'$.

15. Since a is $< b$, there is only one solution, corresponding to the acute value of A.

$\sin A = \dfrac{a \sin B}{b}$. $\quad c = a \sin C \csc A$.

$\log a = 2.3315$	$\log a = 2.3315$
$\operatorname{colog} b = 7.5455$	$\log \sin C = 9.6825$
$\log \sin B = 9.9863$	$\log \csc A = 0.1367$
$\log \sin A = 9.8633$	$\log c = 2.1507$
$\therefore A = 46^\circ 53.3'$,	$\therefore c = 141.48$.

and $C = 180^\circ - 151^\circ 13.3' = 28^\circ 46.7'$.

16. $\sin B = \dfrac{b \sin C}{c}$.

$\log b = 3.4870$
$\operatorname{colog} c = 6.9126$
$\log \sin C = 9.9179$
$\log \sin B = 0.3175$

Since $\log \sin B$ is positive, the triangle is impossible.

17. Since c is $< a$, there is only one solution, corresponding to the acute value of C.

$\sin C = \dfrac{c \sin A}{a}$. $\quad b = a \sin B \csc A$.

$\log c = 9.7086$	$\log a = 9.8511$
$\operatorname{colog} a = 0.1489$	$\log \sin B = 9.9363$
$\log \sin A = 9.7606$	$\log \csc A = 0.2394$
$\log \sin C = 9.6181$	$\log b = 0.0268$
$\therefore C = 24^\circ 31.4'$,	$\therefore b = 1.0637$.

and $B = 180^\circ - 59^\circ 42.4' = 120^\circ 17.6'$.

18. $\sin B = \dfrac{b \sin A}{a}$. $\qquad c = b \sin C$.

$\log b = 2.2206$ $\qquad \log b = 2.2206$

$\operatorname{colog} a = 7.9712$ $\qquad \log \sin C = 9.8843$

$\log \sin A = 9.8082$ $\qquad \log c = 2.1049$

$\log \sin B = 0.0000$ $\qquad \therefore\ c = 127.32.$

$\therefore\ B = 90°$,

and $\qquad C = 90° - 40° 0' 21'' = 49° 59' 39''$.

19. $\sin A = \dfrac{a \sin C}{c}$. $\qquad b_1 = a \sin B_1 \csc A$. $\qquad b_2 = a \sin B_2 \csc A$.

$\log a = 9.5073$ $\qquad \log a = 9.5073$ $\qquad \log a = 9.5073$

$\operatorname{colog} c = 0.5673$ $\qquad \log \sin B_1 = 9.9255$ $\qquad \log \sin B_2 = 9.4853$

$\log \sin C = 9.8989$ $\qquad \log \csc A = 0.0265$ $\qquad \log \csc A = 0.0265$

$\log \sin A = 9.9735$ $\qquad \log b_1 = 9.4593$ $\qquad \log b_2 = 9.0191$

$\therefore\ A_1 = 70° 12'$, $\qquad \therefore\ b_1 = .2879.$ $\qquad \therefore\ b_2 = .1045.$

and $\qquad A_2 = 109° 48'$.

$\therefore\ B_1 = 180° - 122° 36' = 57° 24'$,

and $\qquad B_2 = 180° - 162° 12' = 17° 48'$.

20. Since c is $< b$, there is only one solution, corresponding to the acute value of C.

$\sin C = \dfrac{c \sin B}{b}$. $\qquad a = b \sin A \csc B$.

$\log c = 2.7828$ $\qquad \log b = 2.9092$

$\operatorname{colog} b = 7.0908$ $\qquad \log \sin A = 9.4596$

$\log \sin B = 9.9075$ $\qquad \log \csc B = 0.0925$

$\log \sin C = 9.7811$ $\qquad \log a = 2.4613$

$\therefore\ C = 37° 10'$, $\qquad \therefore\ a = 289.3.$

and $\qquad A = 180° - 163° 15' 20'' = 16° 44' 40''$.

Art. 128. — Page 93.

2. $2K = ac \sin B$.

$\log a = 1.5798$

$\log c = 1.7868$

$\log \sin B = 9.9670$

$\log 2K = 3.3336$

$2K = 2155.7$

$\therefore\ K = 1077.9.$

3. Here $\qquad s = 9$,

$s - a = 4$,

$s - b = 2$,

and $\qquad s - c = 3$.

$K = \sqrt{s(s-a)(s-b)(s-c)}$.

$\log s = 0.9542$
$\log (s-a) = 0.6021$
$\log (s-b) = 0.3010$
$\log (s-c) = 0.4771$
$2)\underline{2.3344}$
$\log K = 1.1672$
$\therefore K = 14.697.$

4. $2K = b^2 \sin C \sin A \csc B.$
$C = 180° - 106° 23' = 73° 37'.$
$2 \log b = 0.6320$
$\log \sin C = 9.9820$
$\log \sin A = 9.9730$
$\log \csc B = 0.2268$
$\log 2K = 0.8138$
$2K = 6.513$
$\therefore K = 3.257.$

5. $2K = bc \sin A.$
$\log b = 2.0649$
$\log c = 2.0000$
$\log \sin A = 9.9449$
$\log 2K = 4.0098$
$2K = 10229$
$\therefore K = 5114.$

6. Here $s = 120,$
$s - a = 41,$
$s - b = 26,$
and $s - c = 53.$
$K = \sqrt{s(s-a)(s-b)(s-c)}.$
$\log s = 2.0792$
$\log (s-a) = 1.6128$
$\log (s-b) = 1.4150$
$\log (s-c) = 1.7243$
$2)\underline{6.8313}$
$\log K = 3.4156$
$\therefore K = 2604.$

7. $2K = a^2 \sin B \sin C \csc A.$
$C = 180° - 67° 8' = 112° 52'.$
$2 \log a = 0.9892$
$\log \sin B = 9.3822$
$\log \sin C = 9.9645$
$\log \csc A = 0.0966$
$\log 2K = 0.4325$
$2K = 2.707$
$\therefore K = 1.353.$

8. $2K = b^2 \sin C \sin A \csc B.$
$B = 180° - 117° 13' = 62° 47'.$
$2 \log b = 9.2850$
$\log \sin C = 9.8132$
$\log \sin A = 9.9880$
$\log \csc B = 0.0510$
$\log 2K = 9.1372$
$2K = .13716$
$\therefore K = .06858.$

9. Here $s = 34,$
$s - a = 10.9,$
$s - b = 14.3,$
and $s - c = 8.8.$
$K = \sqrt{s(s-a)(s-b)(s-c)}.$
$\log s = 1.5315$
$\log (s-a) = 1.0374$
$\log (s-b) = 1.1553$
$\log (s-c) = 0.9445$
$2)\underline{4.6687}$
$\log K = 2.3343$
$\therefore K = 215.9.$

10. $2K = ac \sin B.$
$\log a = 9.5089$
$\log c = 9.9582$
$\log \sin B = 9.9387$
$\log 2K = 9.4058$
$2K = .2546$
$\therefore K = .1273.$

11. $2K = c^2 \sin A \sin B \csc C.$

$A = 180° - 131°49' = 48°11'.$

$2 \log c = 3.8088$

$\log \sin A = 9.8723$

$\log \sin B = 9.9932$

$\log \csc C = 0.2791$

$\log 2K = 3.9534$

$2K = 8982$

$\therefore K = 4491.$

12. $2K = ab \sin C.$

$\log a = 8.0072$

$\log b = 8.2607$

$\log \sin C = 9.2924$

$\log 2K = 5.5603$

$2K = .00003633$

$\therefore K = .00001817.$

13. Here $s = 8.04,$

$s - a = 2.22,$

$s - b = 2.04,$

and $s - c = 3.78.$

$K = \sqrt{s(s-a)(s-b)(s-c)}.$

$\log s = 0.9053$

$\log (s-a) = 0.3464$

$\log (s-b) = 0.3096$

$\log (s-c) = 0.5775$

$2) 2.1388$

$\log K = 1.0694$

$\therefore K = 11.732.$

Art. 129. — Pages 93 and 94.

1. Let A be the first point of observation, and B the second; and let C be the top of the tower, and D its base.

Then $$\frac{AC}{AB} = \frac{\sin ABC}{\sin ACB},$$

or, $$AC = AB \sin ABC \csc ACB$$

$$= 100 \sin 35° 16' \csc 17° 23'.$$

$\log 100 = 2.0000$

$\log \sin 35° 16' = 9.7615$

$\log \csc 17° 23' = 0.5247$

$\therefore \log AC = 2.2862$

Then $CD = AC \sin CAD = AC \sin 52° 39',$

and $AD = AC \cos CAD = AC \cos 52° 39'.$

$\log AC = 2.2862$

$\log \sin 52° 39' = 9.9003$

$\log CD = 2.1865$

$\therefore CD = 153.64.$

$\log AC = 2.2862$

$\log \cos 52° 39' = 9.7830$

$\log AD = 2.0692$

$\therefore AD = 117.27.$

$\therefore BD = AB + AD = 217.27.$

2. Denoting the sides opposite the angles A, B, and C of the triangle ABC by a, b, and c, respectively, we have

$$K = \sqrt{s(s-a)(s-b)(s-c)}.$$

Here $s = 351$, $s - a = 115$, $s - b = 40$, and $s - c = 196$.

$$\begin{aligned} \log s &= 2.5453 \\ \log (s-a) &= 2.0607 \\ \log (s-b) &= 1.6021 \\ \log (s-c) &= 2.2923 \\ \hline 2)\underline{8.5004} \\ \log K &= 4.2502 \\ \therefore K &= 17792. \end{aligned}$$

Denoting the sides opposite the angles A, C, and D of the triangle ACD by a, c, and d, respectively, we have

$$K = \sqrt{s(s-a)(s-c)(s-d)}.$$

Here $s = 334$, $s - a = 82$, $s - c = 229$, and $s - d = 23$.

$$\begin{aligned} \log s &= 2.5237 \\ \log (s-a) &= 1.9138 \\ \log (s-c) &= 2.3598 \\ \log (s-d) &= 1.3617 \\ \hline 2)\underline{8.1590} \\ \log K &= 4.0795 \\ \therefore K &= 12008. \end{aligned}$$

Therefore area $ABCD = 17792 + 12008 = 29800$.

3. Let A be the position of the first post, and B of the second, and let C be the top of the bluff, and D its foot.

Then $$\frac{AC}{AB} = \frac{\sin ABC}{\sin ACB}.$$

Whence,

$$\begin{aligned} CD &= AC \sin CAD \\ &= \frac{AB \sin ABC \sin CAD}{\sin ACB} \\ &= \frac{1000 \sin 9^\circ 33' \sin 27^\circ 40'}{\sin 18^\circ 7'}. \end{aligned}$$

$$\begin{aligned} \log 1000 &= 3.0000 \\ \log \sin 9^\circ 33' &= 9.2198 \\ \log \sin 27^\circ 40' &= 9.6668 \\ \log \csc 18^\circ 7' &= 0.5073 \\ \hline \log CD &= 2.3939 \\ \therefore CD &= 247.7. \end{aligned}$$

4. Let A be the starting-point, and B and C the positions of the first and second yachts, respectively, at the end of 40 minutes.

Then in the triangle ABC, we have

$$AB = 6.96,\ AC = 5.14,\ \text{and}\ \angle A = 45°.$$

$$\tan \tfrac{1}{2}(C - B) = \frac{AB - AC}{AB + AC} \tan \tfrac{1}{2}(C + B).$$

$AB - AC = 1.82$	$\log = 0.2601$
$AB + AC = 12.1$	$\operatorname{colog} = 8.9172$
$\tfrac{1}{2}(C + B) = 67° 30'$	$\log \tan = 0.3828$
	$\log \tan \tfrac{1}{2}(C - B) = 9.5601$
	$\therefore \tfrac{1}{2}(C - B) = 19° 57.5',$

and $B = \tfrac{1}{2}(C + B) - \tfrac{1}{2}(C - B) = 47° 32.5'.$

$$BC = AC \sin A \csc B.$$

$$\begin{aligned} \log AC &= 0.7110 \\ \log \sin A &= 9.8495 \\ \log \csc B &= 0.1321 \\ \log BC &= 0.6926 \\ \therefore\ BC &= 4.927. \end{aligned}$$

5. Let A be the position of the lighthouse, and B and C the first and second positions of the ship.

Then in the triangle ABC, we have

$$BC = 14,\ \angle B = 105°,\ \text{and}\ \angle C = 30°.$$

Whence, $$\angle A = 180° - 135° = 45°.$$

$$AB = BC \sin C \csc A.$$

$$\begin{aligned} \log BC &= 1.1461 \\ \log \sin C &= 9.6990 \\ \log \csc A &= 0.1505 \\ \log AB &= 0.9956 \\ \therefore\ AB &= 9.9. \end{aligned}$$

$$AC = BC \sin B \csc A.$$

$$\begin{aligned} \log BC &= 1.1461 \\ \log \sin B &= 9.9849 \\ \log \csc A &= 0.1505 \\ \log AC &= 1.2815 \\ \therefore\ AC &= 19.122. \end{aligned}$$

6. $$AB = BC \sin C \csc A.$$
$$A = 180^\circ - 158^\circ 23' = 21^\circ 37'.$$
$$\log BC = 2.3187$$
$$\log \sin C = 9.7218$$
$$\log \csc A = 0.4337$$
$$\log AB = 2.4742$$
$$\therefore AB = 298.$$

7. Let A be the point of observation, B the top of the flag-pole, C its foot, and D the base of the tower.

Then, $$\frac{AC}{BC} = \frac{\sin ABC}{\sin BAC},$$

or, $$AC = BC \sin ABC \csc BAC$$
$$= 40 \sin 51^\circ 7' \csc 18^\circ 35'.$$
$$\log 40 = 1.6021$$
$$\log \sin 51^\circ 7' = 9.8912$$
$$\log \csc 18^\circ 35' = 0.4967$$
$$\therefore \log AC = 1.9900$$

$$AD = AC \cos CAD = AC \cos 20^\circ 18'.$$
$$\log AC = 1.9900$$
$$\log \cos 20^\circ 18' = 9.9722$$
$$\log AD = 1.9622$$
$$\therefore AD = 91.66.$$

$$CD = AC \sin CAD = AC \sin 20^\circ 18'.$$
$$\log AC = 1.9900$$
$$\log \sin 20^\circ 18' = 9.5402$$
$$\log CD = 1.5302$$
$$\therefore CD = 33.9.$$

8. $$BC = BD \sin BDC \csc BCD$$
$$= BD \sin 60^\circ \csc 20^\circ.$$

But, $$BD = AD \sin BAD \csc ABD$$
$$= 500 \sin 60^\circ \csc 80^\circ.$$

Whence, $$BC = 500 \sin^2 60^\circ \csc 20^\circ \csc 80^\circ.$$
$$\log 500 = 2.6990$$
$$2 \log \sin 60^\circ = 9.8750$$
$$\log \csc 20^\circ = 0.4659$$
$$\log \csc 80^\circ = 0.0066$$
$$\log BC = 3.0465$$
$$\therefore BC = 1113.1.$$

9. $AC = CD \sin ADC \csc CAD.$

$\log CD = 2.1761$
$\log \sin ADC = 9.6990$
$\log \csc CAD = 0.0866$
$\log AC = 1.9617$
$\therefore AC = 91.56.$

$BC = CD \sin BDC \csc CBD.$

$\log CD = 2.1761$
$\log \sin BDC = 9.9968$
$\log \csc CBD = 0.3430$
$\log BC = 2.5159$
$\therefore BC = 328.$

$$\tan \tfrac{1}{2}(BAC - ABC) = \frac{BC - AC}{BC + AC} \tan \tfrac{1}{2}(BAC + ABC)$$

$BC - AC = 236.44$ $\quad \log = 2.3737$
$BC + AC = 419.56$ $\quad \operatorname{colog} = 7.3772$
$\frac{1}{2}(BAC + ABC) = 77° 30'$ $\quad \log \tan = 0.6542$
$\log \tan \frac{1}{2}(BAC - ABC) = 0.4051$
$\therefore \frac{1}{2}(BAC - ABC) = 68° 31.4',$

and $BAC = \frac{1}{2}(BAC + ABC) + \frac{1}{2}(BAC - ABC) = 146° \ 1.4'.$

$AB = BC \sin ACB \csc BAC.$

$\log BC = 2.5159$
$\log \sin ACB = 9.6259$
$\log \csc BAC = 0.2527$
$\log AB = 2.3945$
$\therefore AB = 248.$

10. Denoting the sides opposite the angles A, B, and C of the triangle ABC by a, b, and c, respectively, we have

$$K = \sqrt{s(s-a)(s-b)(s-c)}.$$

Here, $s = 87.5,$
$s - a = 24.5,$
$s - b = 12.5,$
and $s - c = 50.5.$

$\log s = 1.9420$
$\log (s - a) = 1.3892$
$\log (s - b) = 1.0969$
$\log (s - c) = 1.7033$
$2)\overline{6.1314}$
$\log K = 3.0657$
$\therefore K = 1163.2.$

$$\cos \tfrac{1}{2} BAC = \sqrt{\frac{s(s-a)}{bc}}.$$

$$\begin{aligned} \log s &= 1.9420 \\ \log (s-a) &= 1.3892 \\ \operatorname{colog} b &= 8.1249 \\ \operatorname{colog} c &= 8.4318 \\ \hline 2) & \overline{9.8879} \\ \log \cos \tfrac{1}{2} BAC &= 9.9439 \\ \tfrac{1}{2} BAC &= 28^\circ 30' \\ \therefore\ BAC &= 57^\circ. \end{aligned}$$

Denoting the sides opposite the angles A, B, and D of the triangle ABD by a, b, and d, respectively, we have

$$\cos \tfrac{1}{2} BAD = \sqrt{\frac{s(s-a)}{bd}}.$$

Here, $s = 49.5$,

and $s - a = 7.5$.

$$\begin{aligned} \log s &= 1.6946 \\ \log (s-a) &= 0.8751 \\ \operatorname{colog} b &= 8.6990 \\ \operatorname{colog} d &= 8.4318 \\ \hline 2) & \overline{9.7005} \\ \log \cos \tfrac{1}{2} BAD &= 9.8502 \\ \tfrac{1}{2} BAD &= 44^\circ 54.2' \\ \therefore\ BAD &= 89^\circ 48.4'. \end{aligned}$$

$$\therefore\ CAD = BAD - BAC = 32^\circ 48.4'.$$

$$2 \text{ area } ACD = AC \cdot AD \cdot \sin CAD.$$

$$\begin{aligned} \log AC &= 1.8751 \\ \log AD &= 1.3010 \\ \log \sin CAD &= 9.7339 \\ \hline \log (2 \text{ area } ACD) &= 2.9100 \\ 2 \text{ area } ACD &= 812.8 \\ \therefore \text{ area } ACD &= 406.4. \end{aligned}$$

$$\therefore \text{ area } ABCD = 1163.2 + 406.4 = 1569.6.$$

CHAPTER XI.

Art. 153.—Page 112.

5. $\sin A = \dfrac{\sin a}{\sin c}.$

$\log \sin a = 9.5543$
$\log \sin c = 9.8311$
$\log \sin A = 9.7232$
$180° - A = 31° 55'.$
$\therefore A = 148° 5'.$

$\overset{+}{\cos B} = \dfrac{\overset{-}{\tan a}}{\tan c}.$

$\log \tan a = 9.5842$
$\log \tan c = 9.9646$
$\log \cos B = 9.6196$
$\therefore B = 65° 23.2'.$

$\overset{+}{\cos b} = \dfrac{\overset{-}{\cos c}}{\cos a}.$

$\log \cos c = 9.8665$
$\log \cos a = 9.9702$
$\log \cos b = 9.8963$
$\therefore b = 38° 2'.$

Check.

$\sin A = \dfrac{\cos B}{\cos b}.$

$\log \cos B = 9.6196$
$\log \cos b = 9.8963$
$\log \sin A = 9.7233$

6. $\cos a = \dfrac{\cos A}{\sin B}.$

$\log \cos A = 9.8050$
$\log \sin B = 9.9252$
$\log \cos a = 9.8798$
$\therefore a = 40° 41.8'.$

$\overset{-}{\cos b} = \dfrac{\overset{-}{\cos B}}{\sin A}.$

$\log \cos B = 9.7322$
$\log \sin A = 9.8864$
$\log \cos b = 9.8458$
$180° - b = 45° 29.2'.$
$\therefore b = 134° 30.8'.$

$\overset{-}{\cos c} = \overset{+}{\cot A} \overset{-}{\cot B}.$

$\log \cot A = 9.9187$
$\log \cot B = 9.8070$
$\log \cos c = 9.7257$
$180° - c = 57° 52.5'.$
$\therefore c = 122° 7.5'.$

Check.

$\cos c = \cos a \cos b.$

$\log \cos a = 9.8798$
$\log \cos b = 9.8458$
$\log \cos c = 9.7256$

7. $\tan A = \frac{\tan a}{\sin b}$. (− / −, +)

$\log \tan a = 9.5611$
$\log \sin b = 9.7941$
$\log \tan A = 9.7670$
$180° - A = 30° 19'.$
$\therefore A = 149° 41'.$

$\tan B = \frac{\tan b}{\sin a}$.

$\log \tan b = 9.9006$
$\log \sin a = 9.5341$
$\log \tan B = 0.3665$
$\therefore B = 66° 43.8'.$

$\cos c = \cos a \cos b$. (−, −, +)

$\log \cos a = 9.9730$
$\log \cos b = 9.8935$
$\log \cos c = 9.8665$
$180° - c = 42° 40'.$
$\therefore c = 137° 20'.$

Check.

$\cos c \tan A \tan B = 1.$

$\log \cos c = 9.8665$
$\log \tan A = 9.7670$
$\log \tan B = 0.3665$
$\log 1 = 0.0000$

8. $\sin A = \frac{\cos B}{\cos b}$.

$\log \cos B = 9.2397$
$\log \cos b = 9.5798$
$\log \sin A = 9.6599$
$\therefore A = 27° 11.6',$
or $152° 48.4'.$

$\sin a = \frac{\tan b}{\tan B}$.

$\log \tan b = 0.3864$
$\log \tan B = 0.7537$
$\log \sin a = 9.6327$
$\therefore a = 25° 25.2',$
or $154° 34.8'.$

$\sin c = \frac{\sin b}{\sin B}$.

$\log \sin b = 9.9661$
$\log \sin B = 9.9934$
$\log \sin c = 9.9727$
$\therefore c = 69° 54',$
or $110° 6'.$

Check.

$\sin A = \frac{\sin a}{\sin c}$.

$\log \sin a = 9.6327$
$\log \sin c = 9.9727$
$\log \sin A = 9.6600$

Ans. 1. $A = 27° 11.6'$, $a = 25° 25.2'$, $c = 69° 54'$.
2. $A = 152° 48.4'$, $a = 154° 34.8'$, $c = 110° 6'$.

9. $\tan A = \frac{\cot B}{\cos c}$. (−, +)

$\log \cot B = 9.6064$
$\log \cos c = 9.1525$
$\log \tan A = 0.4539$
$180° - A = 70° 37.5'.$
$\therefore A = 109° 22.5'.$

$\tan a = \cos B \tan c$. (−, −, +)

$\log \cos B = 9.5736$
$\log \tan c = 0.8431$
$\log \tan a = 0.4167$
$180° - a = 69° 2.4'.$
$\therefore a = 110° 57.6'.$

$$\sin b = \sin B \sin c.$$

$\log \sin B = 9.9672$
$\log \sin c = 9.9956$
$\log \sin b = 9.9628$
$180° - b = 66° 38'.$
$\therefore b = 113° 22'.$

Check.

$$\tan A = \frac{\tan a}{\sin b}.$$

$\log \tan a = 0.4167$
$\log \sin b = 9.9628$
$\log \tan A = 0.4539$

10. $\cos A = \cos a \sin B.$

$\log \cos a = 9.6856$
$\log \sin B = 9.9203$
$\log \cos A = 9.6059$
$\therefore A = 66° 12.1'.$

$$\overset{-}{\tan c} = \frac{\overset{+}{\tan a}}{\overset{-}{\cos B}}.$$

$\log \tan a = 0.2562$
$\log \cos B = 9.7438$
$\log \tan c = 0.5124$
$180° - c = 72° 54.9'.$
$\therefore c = 107° 5.1'.$

$$\overset{-}{\tan b} = \overset{+}{\sin a} \overset{-}{\tan B}.$$

$\log \sin a = 9.9418$
$\log \tan B = 0.1765$
$\log \tan b = 0.1183$
$180° - b = 52° 42.6'.$
$\therefore b = 127° 17.4'.$

Check.

$$\cos A = \frac{\tan b}{\tan c}.$$

$\log \tan b = 0.1183$
$\log \tan c = 0.5124$
$\log \cos A = 9.6059$

11. $$\tan A = \frac{\tan a}{\sin b}.$$

$\log \tan a = 0.2683$
$\log \sin b = 9.7675$
$\log \tan A = 0.5008$
$\therefore A = 72° 28.9'.$

$$\overset{-}{\tan B} = \frac{\overset{-}{\tan b}}{\overset{+}{\sin a}}.$$

$\log \tan b = 9.8586$
$\log \sin a = 9.9446$
$\log \tan B = 9.9140$
$180° - B = 39° 21.9'.$
$\therefore B = 140° 38.1'.$

$$\overset{-}{\cos c} = \overset{+}{\cos a} \overset{-}{\cos b}.$$

$\log \cos a = 9.6763$
$\log \cos b = 9.9089$
$\log \cos c = 9.5852$
$180° - c = 67° 22.3'.$
$\therefore c = 112° 37.7'.$

Check.

$$\cos c \tan A \tan B = 1.$$

$\log \cos c = 9.5852$
$\log \tan A = 0.5008$
$\log \tan B = 9.9140$
$\log 1 = 0.0000$

12. $\overset{+}{\sin B} = \dfrac{\overset{-}{\cos A}}{\cos a}$.

$\log \cos A = 9.2324$
$\log \cos a = 9.5736$
$\log \sin B = 9.6588$
$\therefore B = 27^\circ 7.2'$,
or $152^\circ 52.8'$.

$\sin c = \dfrac{\sin a}{\sin A}$.
$\log \sin a = 9.9672$
$\log \sin A = 9.9936$
$\log \sin c = 9.9736$
$\therefore c = 70^\circ 14'$,
or $109^\circ 46'$.

$\overset{+}{\sin b} = \dfrac{\overset{-}{\tan a}}{\tan A}$.

$\log \tan a = 0.3936$
$\log \tan A = 0.7611$
$\log \sin b = 9.6325$
$\therefore b = 25^\circ 24.4'$,
or $154^\circ 35.6'$.

Check.

$\sin B = \dfrac{\sin b}{\sin c}$.
$\log \sin b = 9.6325$
$\log \sin c = 9.9736$
$\log \sin B = 9.6589$

Ans. 1. $B = 27^\circ\ 7.2'$, $b = 25^\circ 24.4'$, $c = 109^\circ 46'$.
2. $B = 152^\circ 52.8'$, $b = 154^\circ 35.6'$, $c = 70^\circ 14'$.

13. $\overset{-}{\cos A} = \dfrac{\overset{+}{\tan b}}{\tan c}$.

$\log \tan b = 9.4281$
$\log \tan c = 9.7196$
$\log \cos A = 9.7085$
$180^\circ - A = 59^\circ 15.7'$.
$\therefore A = 120^\circ 44.3'$.

$\sin B = \dfrac{\sin b}{\sin c}$.
$\log \sin b = 9.4130$
$\log \sin c = 9.6668$
$\log \sin B = 9.7462$
$\therefore B = 33^\circ 52.6'$.

Check.

$\overset{-}{\cos a} = \dfrac{\overset{-}{\cos c}}{\cos b}$.

$\log \cos c = 9.9473$
$\log \cos b = 9.9849$
$\log \cos a = 9.9624$
$180^\circ - a = 23^\circ 30'$.
$\therefore a = 156^\circ 30'$.

$\sin B = \dfrac{\cos A}{\cos a}$.
$\log \cos A = 9.7085$
$\log \cos a = 9.9624$
$\log \sin B = 9.7461$

14. $\cos a = \dfrac{\cos A}{\sin B}$.

$\log \cos A = 9.6573$
$\log \sin B = 9.7801$
$\log \cos a = 9.8772$
$\therefore a = 41^\circ 5.5'$.

$\cos b = \dfrac{\cos B}{\sin A}$.

$\log \cos B = 9.9019$
$\log \sin A = 9.9498$
$\log \cos b = 9.9521$
$\therefore b = 26^\circ 25'$.

$\cos c = \cot A \cot B$.

$\log \cot A = 9.7075$
$\log \cot B = 0.1219$
$\log \cos c = 9.8294$
$\therefore c = 47^\circ 32.1'$.

Check.

$\cos c = \cos a \cos b$.

$\log \cos a = 9.8772$
$\log \cos b = 9.9521$
$\log \cos c = 9.8293$

15. $\sin a = \sin A \sin c$.

$\log \sin A = 9.9809$
$\log \sin c = 9.9589$
$\log \sin a = 9.9398$
$\therefore a = 60^\circ 31.4'$.

$\overset{-}{\tan} B = \dfrac{\overset{+}{\cot} A}{\underset{-}{\cos} c}$.

$\log \cot A = 9.4822$
$\log \cos c = 9.6183$
$\log \tan B = 9.8639$
$180^\circ - B = 36^\circ 10'$.
$\therefore B = 143^\circ 50'$.

$\overset{-}{\tan} b = \overset{+}{\cos} A \tan c$.

$\log \cos A = 9.4630$
$\log \tan c = 0.3406$
$\log \tan b = 9.8036$
$180^\circ - b = 32^\circ 27.9'$.
$\therefore b = 147^\circ 32.1'$.

Check.

$\tan B = \dfrac{\tan b}{\sin a}$.

$\log \tan b = 9.8036$
$\log \sin a = 9.9398$
$\log \tan B = 9.8638$

16. $\overset{+}{\sin} A = \dfrac{\overset{-}{\cos} B}{\underset{-}{\cos} b}$.

$\log \cos B = 9.9129$
$\log \cos b = 9.9213$
$\log \sin A = 9.9916$
$\therefore A = 78^\circ 46.7'$,
or $101^\circ 13.3'$.

$\overset{+}{\sin} a = \dfrac{\overset{-}{\tan} b}{\underset{-}{\tan} B}$.

$\log \tan b = 9.8202$
$\log \tan B = 9.8468$
$\log \sin a = 9.9734$
$\therefore a = 70^\circ 10'$,
or $109^\circ 50'$.

$$\sin c = \frac{\sin b}{\sin B}.$$

$\log \sin b = 9.7415$
$\log \sin B = 9.7597$
$\log \sin c = 9.9818$
$\therefore c = 73^\circ 32.5'$,
or $106^\circ 27.5'$.

Check.

$$\sin A = \frac{\sin a}{\sin c}.$$

$\log \sin a = 9.9734$
$\log \sin c = 9.9818$
$\log \sin A = 9.9916$

Ans. 1. $A = 78^\circ 46.7'$, $a = 70^\circ 10'$, $c = 106^\circ 27.5'$.
2. $A = 101^\circ 13.3'$, $a = 109^\circ 50'$, $c = 73^\circ 32.5'$.

17.

$$\tan A = \frac{\cot B}{\cos c}.$$

$\log \cot B = 9.5998$
$\log \cos c = 9.8291$
$\log \tan A = 9.7707$
$\therefore A = 30^\circ 32.1'$.

$$\sin b = \sin B \sin c.$$

$\log \sin B = 9.9681$
$\log \sin c = 9.8681$
$\log \sin b = 9.8362$
$\therefore b = 43^\circ 17.9'$.

$$\tan a = \cos B \tan c.$$

$\log \cos B = 9.5679$
$\log \tan c = 0.0389$
$\log \tan a = 9.6068$
$\therefore a = 22^\circ 1.1'$.

Check.

$$\tan A = \frac{\tan a}{\sin b}.$$

$\log \tan a = 9.6068$
$\log \sin b = 9.8362$
$\log \tan A = 9.7706$

18.

$$\tan a = \tan A \sin b.$$

$\log \tan A = 9.5152$
$\log \sin b = 9.8769$
$\log \tan a = 9.3921$
$180^\circ - a = 13^\circ 51.3'$.
$\therefore a = 166^\circ 8.7'$.

$$\tan c = \frac{\tan b}{\cos A}.$$

$\log \tan b = 0.0588$
$\log \cos A = 9.9779$
$\log \tan c = 0.0809$
$\therefore c = 50^\circ 18.4'$.

$$\cos B = \sin A \cos b.$$

$\log \sin A = 9.4931$
$\log \cos b = 9.8181$
$\log \cos B = 9.3112$
$180^\circ - B = 78^\circ 11.1'$.
$\therefore B = 101^\circ 48.9'$.

Check.

$$\cos B = \frac{\tan a}{\tan c}.$$

$\log \tan a = 9.3921$
$\log \tan c = 0.0809$
$\log \cos B = 9.3112$

19.

$$\tan A = \frac{\tan a}{\sin b}. \qquad \tan B = \frac{\tan b}{\sin a}.$$

$$\log \tan a = 0.3634 \qquad \log \tan b = 0.4207$$
$$\log \sin b = 9.9707 \qquad \log \sin a = 9.9626$$
$$\log \tan A = 0.3927 \qquad \log \tan B = 0.4581$$
$$180^\circ - A = 67^\circ 57.5'. \qquad 180^\circ - B = 70^\circ 48'.$$
$$\therefore A = 112^\circ 2.5'. \qquad \therefore B = 109^\circ 12'.$$

$$\cos c = \cos a \cos b.$$

$$\log \cos a = 9.5993$$
$$\log \cos b = 9.5500$$
$$\log \cos c = 9.1493$$
$$\therefore c = 81^\circ 53.6'.$$

Check.

$$\cos c \tan A \tan B = 1.$$

$$\log \cos c = 9.1493$$
$$\log \tan A = 0.3927$$
$$\log \tan B = 0.4581$$
$$\log 1 = 0.0001$$

20.

$$\cos A = \cos a \sin B. \qquad \tan c = \frac{\tan a}{\cos B}.$$

$$\log \cos a = 9.8652 \qquad \log \tan a = 9.9674$$
$$\log \sin B = 9.9846 \qquad \log \cos B = 9.4172$$
$$\log \cos A = 9.8498 \qquad \log \tan c = 0.5502$$
$$180^\circ - A = 44^\circ 57.5'. \qquad 180^\circ - c = 74^\circ 16'.$$
$$\therefore A = 135^\circ 2.5'. \qquad \therefore c = 105^\circ 44'.$$

$$\tan b = \sin a \tan B.$$

$$\log \sin a = 9.8325$$
$$\log \tan B = 0.5674$$
$$\log \tan b = 0.3999$$
$$\therefore b = 68^\circ 17.3'.$$

Check.

$$\cos A = \frac{\tan b}{\tan c}.$$

$$\log \tan b = 0.3999$$
$$\log \tan c = 0.5502$$
$$\log \cos A = 9.8497$$

21.

$$\cos a = \frac{\cos A}{\sin B}. \qquad \cos b = \frac{\cos B}{\sin A}.$$

$$\log \cos A = 9.9129 \qquad \log \cos B = 9.2896$$
$$\log \sin B = 9.9916 \qquad \log \sin A = 9.7597$$
$$\log \cos a = 9.9213 \qquad \log \cos b = 9.5299$$
$$180^\circ - a = 33^\circ 27.5'. \qquad 180^\circ - b = 70^\circ 11.9'.$$
$$\therefore a = 146^\circ 32.5'. \qquad \therefore b = 109^\circ 48.1'.$$

$$\overset{+}{\cos c} = \overset{-}{\cot A}\,\overset{-}{\cot B}.$$

$\log \cot A = 0.1532$
$\log \cot B = 9.2980$
$\log \cos c = 9.4512$
$\therefore\ c = 73^\circ 35'.$

Check.

$\cos c = \cos a \cos b.$
$\log \cos a = 9.9213$
$\log \cos b = 9.5299$
$\log \cos c = 9.4512$

22. $\sin A = \dfrac{\sin a}{\sin c}.$

$\log \sin a = 9.9710$
$\log \sin c = 9.9980$
$\log \sin A = 9.9730$
$\therefore\ A = 70^\circ.$

$\cos b = \dfrac{\cos c}{\cos a}.$

$\log \cos c = 8.9855$
$\log \cos a = 9.5484$
$\log \cos b = 9.4371$
$\therefore\ b = 74^\circ 7.3'.$

$\cos B = \dfrac{\tan a}{\tan c}.$

$\log \tan a = 0.4226$
$\log \tan c = 1.0125$
$\log \cos B = 9.4101$
$\therefore\ B = 75^\circ 6.2'.$

Check.

$\sin A = \dfrac{\cos B}{\cos b}.$

$\log \cos B = 9.4101$
$\log \cos b = 9.4371$
$\log \sin A = 9.9730$

Art. 154. — Page 113.

2. In the polar triangle, $a' = 41^\circ$ and $B' = 37^\circ$.

$\cos A' = \cos a' \sin B'.$

$\log \cos a' = 9.8778$
$\log \sin B' = 9.7795$
$\log \cos A' = 9.6573$
$\therefore\ A' = 62^\circ 58.8'.$

$\tan b' = \sin a' \tan B'.$

$\log \sin a' = 9.8169$
$\log \tan B' = 9.8771$
$\log \tan b' = 9.6940$
$\therefore\ b' = 26^\circ 18.1'.$

$\tan c' = \dfrac{\tan a'}{\cos B'}.$

$\log \tan a' = 9.9392$
$\log \cos B' = 9.9023$
$\log \tan c' = 0.0369$
$\therefore\ c' = 47^\circ 26'.$

Check.

$\cos A' = \dfrac{\tan b'}{\tan c'}.$

$\log \tan b' = 9.6940$
$\log \tan c' = 0.0369$
$\log \cos A' = 9.6571$

$\therefore$ in the quadrantal triangle,

$a = 117^\circ 1.2'$, $B = 153^\circ 41.9'$, and $C = 132^\circ 34'$.

3. In the polar triangle, $a' = 134^\circ 30'$ and $b' = 40^\circ 40'$.

$$\tan A' = \frac{\tan a'}{\sin b'}.$$

$$\begin{aligned} \log \tan a' &= 0.0076 \\ \log \sin b' &= 9.8140 \\ \log \tan A' &= 0.1936 \\ \therefore\ 180^\circ - A' &= 57^\circ 22.1'. \end{aligned}$$

$$\tan B' = \frac{\tan b'}{\sin a'}.$$

$$\begin{aligned} \log \tan b' &= 9.9341 \\ \log \sin a' &= 9.8532 \\ \log \tan B' &= 0.0809 \\ \therefore\ B' &= 50^\circ 18.4'. \end{aligned}$$

$$\cos c' = \cos a' \cos b'.$$

$$\begin{aligned} \log \cos a' &= 9.8457 \\ \log \cos b' &= 9.8800 \\ \log \cos c' &= 9.7257 \\ \therefore\ 180^\circ - c' &= 57^\circ 52.5'. \end{aligned}$$

Check.

$$\cos c' \tan A' \tan B' = 1.$$

$$\begin{aligned} \log \cos c' &= 9.7257 \\ \log \tan A' &= 0.1936 \\ \log \tan B' &= 0.0809 \\ \log 1 &= 0.0002 \end{aligned}$$

$\therefore$ in the quadrantal triangle,

$$a = 57^\circ 22.1',\ b = 129^\circ 41.6',\ \text{and}\ C = 57^\circ 52.5'.$$

4. In the polar triangle, $A' = 149^\circ 40'$ and $c' = 137^\circ 20'$.

$$\sin a' = \sin A' \sin c'.$$

$$\begin{aligned} \log \sin A' &= 9.7033 \\ \log \sin c' &= 9.8311 \\ \log \sin a' &= 9.5344 \\ \therefore\ 180^\circ - a' &= 20^\circ 0.9'. \end{aligned}$$

$$\tan B' = \frac{\cot A'}{\cos c'}.$$

$$\begin{aligned} \log \cot A' &= 0.2327 \\ \log \cos c' &= 9.8665 \\ \log \tan B' &= 0.3662 \\ \therefore\ B' &= 66^\circ 42.9'. \end{aligned}$$

$$\tan b' = \cos A' \tan c'.$$

$$\begin{aligned} \log \cos A' &= 9.9361 \\ \log \tan c' &= 9.9646 \\ \log \tan b' &= 9.9007 \\ \therefore\ b' &= 38^\circ 30.4'. \end{aligned}$$

Check.

$$\tan B' = \frac{\tan b'}{\sin a'}.$$

$$\begin{aligned} \log \tan b' &= 9.9007 \\ \log \sin a' &= 9.5344 \\ \log \tan B' &= 0.3663 \end{aligned}$$

$\therefore$ in the quadrantal triangle,

$$A = 20^\circ 0.9',\ b = 113^\circ 17.1',\ \text{and}\ B = 141^\circ 29.6'.$$

5. In the polar triangle, $b' = 109° 48'$ and $c' = 73° 35'$.

$$\cos a' = \frac{\cos c'}{\cos b'}. \qquad \cos A' = \frac{\tan b'}{\tan c'}.$$

$\log \cos c' = 9.4512$ $\log \tan b' = 0.4437$

$\log \cos b' = 9.5299$ $\log \tan c' = 0.5307$

$\log \cos a' = 9.9213$ $\log \cos A' = 9.9130$

$\therefore 180° - a' = 33° 27.5'.$ $\therefore 180° - A = 35° 4.4'.$

$$\sin B' = \frac{\sin b'}{\sin c'}.$$

$\log \sin b' = 9.9735$

$\log \sin c' = 9.9819$

$\log \sin B' = 9.9916$

$\therefore 180° - B' = 78° 46.7'.$

Check.

$$\sin B' = \frac{\cos A'}{\cos a'}.$$

$\log \cos A' = 9.9130$

$\log \cos a = 9.9213$

$\log \sin B' = 9.9917$

$\therefore$ in the quadrantal triangle,

$a = 35° 4.4'$, $A = 33° 27.5'$, and $b = 78° 46.7'$.

6. In the polar triangle, $a' = 74° 7'$ and $A' = 75° 6'$.

$$\sin b' = \frac{\tan a'}{\tan A'}. \qquad \sin B' = \frac{\cos A'}{\cos a'}.$$

$\log \tan a' = 0.5459$ $\log \cos A' = 9.4102$

$\log \tan A' = 0.5750$ $\log \cos a' = 9.4372$

$\log \sin b' = 9.9709$ $\log \sin B' = 9.9730$

$\therefore b' = 69° 16'$, $\therefore B' = 70°$,

or $110° 44'$. or $110°$.

$$\sin c' = \frac{\sin a'}{\sin A'}.$$

$\log \sin a' = 9.9831$

$\log \sin A' = 9.9851$

$\log \sin c' = 9.9980$

$\therefore c' = 84° 30'$,

or $95° 30'$.

Check.

$$\sin B' = \frac{\sin b'}{\sin c'}.$$

$\log \sin b' = 9.9709$

$\log \sin c' = 9.9980$

$\log \sin B' = 9.9729$

$\therefore$ in the quadrantal triangle,

1. $B = 69° 16'$, $b = 70°$, and $C = 84° 30'$.
2. $B = 110° 44'$, $b = 110°$, and $C = 95° 30'$.

CHAPTER XII.

Art. 167. — Page 125.

2. Here $\frac{1}{2}(A-B) = 18^\circ 30'$, $\frac{1}{2}(A+B) = 59^\circ 30'$, $\frac{1}{2}c = 54^\circ$.

$$\tan\tfrac{1}{2}(a-b) = \frac{\sin\frac{1}{2}(A-B)}{\sin\frac{1}{2}(A+B)}\tan\tfrac{1}{2}c. \qquad \tan\tfrac{1}{2}(a+b) = \frac{\cos\frac{1}{2}(A-B)}{\cos\frac{1}{2}(A+B)}\tan\tfrac{1}{2}c.$$

$$\begin{aligned} \log\sin\tfrac{1}{2}(A-B) &= 9.5015 \\ \log\csc\tfrac{1}{2}(A+B) &= 0.0647 \\ \log\tan\tfrac{1}{2}c &= 0.1387 \\ \log\tan\tfrac{1}{2}(a-b) &= 9.7049 \end{aligned} \qquad \begin{aligned} \log\cos\tfrac{1}{2}(A-B) &= 9.9770 \\ \log\sec\tfrac{1}{2}(A+B) &= 0.2945 \\ \log\tan\tfrac{1}{2}c &= 0.1387 \\ \log\tan\tfrac{1}{2}(a+b) &= 0.4102 \end{aligned}$$

$\therefore \frac{1}{2}(a-b) = 26^\circ 52.8'.$ $\qquad \therefore \frac{1}{2}(a+b) = 68^\circ 45'.$

$$\therefore a = \tfrac{1}{2}(a+b) + \tfrac{1}{2}(a-b) = 95^\circ 37.8',$$

and $$b = \tfrac{1}{2}(a+b) - \tfrac{1}{2}(a-b) = 41^\circ 52.2'.$$

$$\cot\tfrac{1}{2}C = \frac{\sin\frac{1}{2}(a+b)}{\sin\frac{1}{2}(a-b)}\tan\tfrac{1}{2}(A-B).$$

$$\begin{aligned} \log\sin\tfrac{1}{2}(a+b) &= 9.9694 \\ \log\csc\tfrac{1}{2}(a-b) &= 0.3447 \\ \log\tan\tfrac{1}{2}(A-B) &= 9.5245 \\ \log\cot\tfrac{1}{2}C &= 9.8386 \end{aligned}$$

$\frac{1}{2}C = 55^\circ 24.4'$

$\therefore C = 110^\circ 48.8'.$

3. Here $\frac{1}{2}(B-C) = 42^\circ 30'$, $\frac{1}{2}(B+C) = 92^\circ 30'$, $\frac{1}{2}a = 35^\circ 10'$.

$$\tan\tfrac{1}{2}(b-c) = \frac{\sin\frac{1}{2}(B-C)}{\sin\frac{1}{2}(B+C)}\tan\tfrac{1}{2}a. \qquad \tan\tfrac{1}{2}(b+c) = \frac{\cos\frac{1}{2}(B-C)}{\cos\frac{1}{2}(B+C)}\tan\tfrac{1}{2}a.$$

$$\begin{aligned} \log\sin\tfrac{1}{2}(B-C) &= 9.8297 \\ \log\csc\tfrac{1}{2}(B+C) &= 0.0004 \\ \log\tan\tfrac{1}{2}a &= 9.8479 \\ \log\tan\tfrac{1}{2}(b-c) &= 9.6780 \end{aligned} \qquad \begin{aligned} \log\cos\tfrac{1}{2}(B-C) &= 9.8676 \\ \log\sec\tfrac{1}{2}(B+C) &= 1.3603 \\ \log\tan\tfrac{1}{2}a &= 9.8479 \\ \log\tan\tfrac{1}{2}(b+c) &= 1.0758 \end{aligned}$$

$\therefore \frac{1}{2}(b-c) = 25^\circ 28.5'.$ $\qquad 180^\circ - \frac{1}{2}(b+c) = 85^\circ 11.9'$

$\therefore \frac{1}{2}(b+c) = 94^\circ 48.1'.$

$$\therefore b = \tfrac{1}{2}(b+c) + \tfrac{1}{2}(b-c) = 120^\circ 16.6',$$

and $$c = \tfrac{1}{2}(b+c) - \tfrac{1}{2}(b-c) = 69^\circ 19.6'.$$

$$\cot \tfrac{1}{2}A = \frac{\sin \tfrac{1}{2}(b+c)}{\sin \tfrac{1}{2}(b-c)} \tan \tfrac{1}{2}(B-C).$$

$\log \sin \tfrac{1}{2}(b+c) = 9.9985$
$\log \csc \tfrac{1}{2}(b-c) = 0.3664$
$\log \tan \tfrac{1}{2}(B-C) = \underline{9.9621}$
$\log \cot \tfrac{1}{2}A = \overline{0.3270}$
$\tfrac{1}{2}A = 25^\circ 13.1'$
$\therefore A = 50^\circ 26.2'.$

4. Here $\tfrac{1}{2}(C-A) = 45^\circ 20'$, $\tfrac{1}{2}(C+A) = 77^\circ$, $\tfrac{1}{2}b = 20^\circ 20'$.

$$\tan \tfrac{1}{2}(c-a) = \frac{\sin \tfrac{1}{2}(C-A)}{\sin \tfrac{1}{2}(C+A)} \tan \tfrac{1}{2}b. \qquad \tan \tfrac{1}{2}(c+a) = \frac{\cos \tfrac{1}{2}(C-A)}{\cos \tfrac{1}{2}(C+A)} \tan \tfrac{1}{2}b.$$

$\log \sin \tfrac{1}{2}(C-A) = 9.8520$
$\log \csc \tfrac{1}{2}(C+A) = 0.0113$
$\log \tan \tfrac{1}{2}b = \underline{9.5689}$
$\log \tan \tfrac{1}{2}(c-a) = \overline{9.4322}$
$\therefore \tfrac{1}{2}(c-a) = 15^\circ 8.2'.$

$\log \cos \tfrac{1}{2}(C-A) = 9.8469$
$\log \sec \tfrac{1}{2}(C+A) = 0.6479$
$\log \tan \tfrac{1}{2}b = \underline{9.5689}$
$\log \tan \tfrac{1}{2}(c+a) = \overline{0.0637}$
$\therefore \tfrac{1}{2}(c+a) = 49^\circ 11.2'.$

$\therefore a = \tfrac{1}{2}(c+a) - \tfrac{1}{2}(c-a) = 34^\circ 3',$

and $c = \tfrac{1}{2}(c+a) + \tfrac{1}{2}(c-a) = 64^\circ 19.4'.$

$$\cot \tfrac{1}{2}B = \frac{\sin \tfrac{1}{2}(c+a)}{\sin \tfrac{1}{2}(c-a)} \tan \tfrac{1}{2}(C-A).$$

$\log \sin \tfrac{1}{2}(c+a) = 9.8790$
$\log \csc \tfrac{1}{2}(c-a) = 0.5831$
$\log \tan \tfrac{1}{2}(C-A) = \underline{0.0051}$
$\log \cot \tfrac{1}{2}B = \overline{0.4672}$
$\tfrac{1}{2}B = 18^\circ 49.8'$
$\therefore B = 37^\circ 39.6'.$

5. Here $\tfrac{1}{2}(B-A) = 18^\circ 47'$, $\tfrac{1}{2}(B+A) = 126^\circ 59'$, $\tfrac{1}{2}c = 63^\circ 16'$.

$$\tan \tfrac{1}{2}(b-a) = \frac{\sin \tfrac{1}{2}(B-A)}{\sin \tfrac{1}{2}(B+A)} \tan \tfrac{1}{2}c. \qquad \overset{-}{\underset{+}{\tan}} \tfrac{1}{2}(b+a) = \overset{+}{\underset{-}{\frac{\cos \tfrac{1}{2}(B-A)}{\cos \tfrac{1}{2}(B+A)}}} \tan \tfrac{1}{2}c.$$

$\log \sin \tfrac{1}{2}(B-A) = 9.5079$
$\log \csc \tfrac{1}{2}(B+A) = 0.0976$
$\log \tan \tfrac{1}{2}c = \underline{0.2979}$
$\log \tan \tfrac{1}{2}(b-a) = \overline{9.9034}$
$\therefore \tfrac{1}{2}(b-a) = 38^\circ 40.8'.$

$\log \cos \tfrac{1}{2}(B-A) = 9.9762$
$\log \sec \tfrac{1}{2}(B+A) = 0.2207$
$\log \tan \tfrac{1}{2}c = \underline{0.2979}$
$\log \tan \tfrac{1}{2}(b+a) = \overline{0.4948}$
$180^\circ - \tfrac{1}{2}(b+a) = 72^\circ 15.2'$
$\therefore \tfrac{1}{2}(b+a) = 107^\circ 44.8'.$

$\therefore a = \tfrac{1}{2}(b+a) - \tfrac{1}{2}(b-a) = 69^\circ 4',$

and $b = \tfrac{1}{2}(b+a) + \tfrac{1}{2}(b-a) = 146^\circ 25.6'.$

$$\cot \tfrac{1}{2} C = \frac{\sin \tfrac{1}{2}(b+a)}{\sin \tfrac{1}{2}(b-a)} \tan \tfrac{1}{2}(B-A).$$

$\log \sin \tfrac{1}{2}(b+a) = 9.9788$
$\log \csc \tfrac{1}{2}(b-a) = 0.2042$
$\log \tan \tfrac{1}{2}(B-A) = 9.5316$
$\log \cot \tfrac{1}{2} C = 9.7146$
$\tfrac{1}{2} C = 62^\circ 36.1'$
$\therefore\ C = 125^\circ 12.2'.$

Art. 168. — Page 126.

2. Here $\tfrac{1}{2}(a-b) = 12^\circ 30'$, $\tfrac{1}{2}(a+b) = 59^\circ 30'$, $\tfrac{1}{2} C = 16^\circ 30'$.

$$\tan \tfrac{1}{2}(A-B) = \frac{\sin \tfrac{1}{2}(a-b)}{\sin \tfrac{1}{2}(a+b)} \cot \tfrac{1}{2} C. \qquad \tan \tfrac{1}{2}(A+B) = \frac{\cos \tfrac{1}{2}(a-b)}{\cos \tfrac{1}{2}(a+b)} \cot \tfrac{1}{2} C.$$

$\log \sin \tfrac{1}{2}(a-b) = 9.3353$ $\qquad$ $\log \cos \tfrac{1}{2}(a-b) = 9.9896$
$\log \csc \tfrac{1}{2}(a+b) = 0.0647$ $\qquad$ $\log \sec \tfrac{1}{2}(a+b) = 0.2945$
$\log \cot \tfrac{1}{2} C = 0.5284$ $\qquad$ $\log \cot \tfrac{1}{2} C = 0.5284$
$\log \tan \tfrac{1}{2}(A-B) = 9.9284$ $\qquad$ $\log \tan \tfrac{1}{2}(A+B) = 0.8125$
$\therefore\ \tfrac{1}{2}(A-B) = 40^\circ 18'.$ $\qquad$ $\therefore\ \tfrac{1}{2}(A+B) = 81^\circ 14.8'.$

$$\therefore\ A = \tfrac{1}{2}(A+B) + \tfrac{1}{2}(A-B) = 121^\circ 32.8',$$

and $$B = \tfrac{1}{2}(A+B) - \tfrac{1}{2}(A-B) = 40^\circ 56.8'.$$

$$\tan \tfrac{1}{2} c = \frac{\sin \tfrac{1}{2}(A+B)}{\sin \tfrac{1}{2}(A-B)} \tan \tfrac{1}{2}(a-b).$$

$\log \sin \tfrac{1}{2}(A+B) = 9.9949$
$\log \csc \tfrac{1}{2}(A-B) = 0.1892$
$\log \tan \tfrac{1}{2}(a-b) = 9.3458$
$\log \tan \tfrac{1}{2} c = 9.5299$
$\tfrac{1}{2} c = 18^\circ 42.9'$
$\therefore\ c = 37^\circ 25.8'.$

3. Here $\tfrac{1}{2}(a-c) = 19^\circ$, $\tfrac{1}{2}(a+c) = 79^\circ$, $\tfrac{1}{2} B = 55^\circ$.

$$\tan \tfrac{1}{2}(A-C) = \frac{\sin \tfrac{1}{2}(a-c)}{\sin \tfrac{1}{2}(a+c)} \cot \tfrac{1}{2} B. \qquad \tan \tfrac{1}{2}(A+C) = \frac{\cos \tfrac{1}{2}(a-c)}{\cos \tfrac{1}{2}(a+c)} \cot \tfrac{1}{2} B.$$

$\log \sin \tfrac{1}{2}(a-c) = 9.5126$ $\qquad$ $\log \cos \tfrac{1}{2}(a-c) = 9.9757$
$\log \csc \tfrac{1}{2}(a+c) = 0.0081$ $\qquad$ $\log \sec \tfrac{1}{2}(a+c) = 0.7194$
$\log \cot \tfrac{1}{2} B = 9.8452$ $\qquad$ $\log \cot \tfrac{1}{2} B = 9.8452$
$\log \tan \tfrac{1}{2}(A-C) = 9.3659$ $\qquad$ $\log \tan \tfrac{1}{2}(A+C) = 0.5403$
$\therefore\ \tfrac{1}{2}(A-C) = 13^\circ 4.4'.$ $\qquad$ $\therefore\ \tfrac{1}{2}(A+C) = 73^\circ 55.3'.$

$$\therefore\ A = \tfrac{1}{2}(A+C) + \tfrac{1}{2}(A-C) = 86^\circ 59.7',$$

and $$C = \tfrac{1}{2}(A+C) - \tfrac{1}{2}(A-C) = 60^\circ 50.9'.$$

$$\tan \tfrac{1}{2} b = \frac{\sin \tfrac{1}{2}(A + C)}{\sin \tfrac{1}{2}(A - C)} \tan \tfrac{1}{2}(a - c).$$

$\log \sin \tfrac{1}{2}(A + C) = 9.9827$
$\log \csc \tfrac{1}{2}(A - C) = 0.6455$
$\log \tan \tfrac{1}{2}(a - c) = \underline{9.5370}$
$\log \tan \tfrac{1}{2} b = \overline{0.1652}$
$\tfrac{1}{2} b = 55° 38.5'$
$\therefore b = 111° 17'.$

4. Here $\tfrac{1}{2}(b - c) = 24° 50'$, $\tfrac{1}{2}(b + c) = 95° 30'$, $\tfrac{1}{2} A = 25°$.

$$\tan \tfrac{1}{2}(B - C) = \frac{\sin \tfrac{1}{2}(b - c)}{\sin \tfrac{1}{2}(b + c)} \cot \tfrac{1}{2} A. \qquad \tan \tfrac{1}{2}(B + C) = \frac{\cos \tfrac{1}{2}(b - c)}{\cos \tfrac{1}{2}(b + c)} \cot \tfrac{1}{2} A.$$

$\log \sin \tfrac{1}{2}(b - c) = 9.6232$
$\log \csc \tfrac{1}{2}(b + c) = 0.0020$
$\log \cot \tfrac{1}{2} A = \underline{0.3313}$
$\log \tan \tfrac{1}{2}(B - C) = \overline{9.9565}$
$\therefore \tfrac{1}{2}(B - C) = 42° 8.1'.$

$\log \cos \tfrac{1}{2}(b - c) = 9.9579$
$\log \sec \tfrac{1}{2}(b + c) = 1.0184$
$\log \cot \tfrac{1}{2} A = \underline{0.3313}$
$\log \tan \tfrac{1}{2}(B + C) = \overline{1.3076}$
$180° - \tfrac{1}{2}(B + C) = 87° 10.8'$
$\therefore \tfrac{1}{2}(B + C) = 92° 49.2'.$

$\therefore B = \tfrac{1}{2}(B + C) + \tfrac{1}{2}(B - C) = 134° 57.3',$
and $C = \tfrac{1}{2}(B + C) - \tfrac{1}{2}(B - C) = 50° 41.1'.$

$$\tan \tfrac{1}{2} a = \frac{\sin \tfrac{1}{2}(B + C)}{\sin \tfrac{1}{2}(B - C)} \tan \tfrac{1}{2}(b - c).$$

$\log \sin \tfrac{1}{2}(B + C) = 9.9995$
$\log \csc \tfrac{1}{2}(B - C) = 0.1734$
$\log \tan \tfrac{1}{2}(b - c) = \underline{9.6664}$
$\log \tan \tfrac{1}{2} a = \overline{9.8393}$
$\tfrac{1}{2} a = 34° 34.4'$
$\therefore a = 69° 8.8'.$

5. Here $\tfrac{1}{2}(b - a) = 14° 20'$, $\tfrac{1}{2}(b + a) = 139° 30'$, $\tfrac{1}{2} C = 70° 10'$.

$$\tan \tfrac{1}{2}(B - A) = \frac{\sin \tfrac{1}{2}(b - a)}{\sin \tfrac{1}{2}(b + a)} \cot \tfrac{1}{2} C. \qquad \tan \tfrac{1}{2}(B + A) = \frac{\cos \tfrac{1}{2}(b - a)}{\cos \tfrac{1}{2}(b + a)} \cot \tfrac{1}{2} C.$$

$\log \sin \tfrac{1}{2}(b - a) = 9.3937$
$\log \csc \tfrac{1}{2}(b + a) = 0.1875$
$\log \cot \tfrac{1}{2} C = \underline{9.5571}$
$\log \tan \tfrac{1}{2}(B - A) = \overline{9.1383}$
$\therefore \tfrac{1}{2}(B - A) = 7° 49.8'.$

$\log \cos \tfrac{1}{2}(b - a) = 9.9863$
$\log \sec \tfrac{1}{2}(b + a) = 0.1190$
$\log \cot \tfrac{1}{2} C = \underline{9.5571}$
$\log \tan \tfrac{1}{2}(B + A) = \overline{9.6624}$
$180° - \tfrac{1}{2}(B + A) = 24° 41.2'$
$\therefore \tfrac{1}{2}(B + A) = 155° 18.8'.$

$\therefore A = \tfrac{1}{2}(B + A) - \tfrac{1}{2}(B - A) = 147° 29',$
and $B = \tfrac{1}{2}(B + A) + \tfrac{1}{2}(B - A) = 163° 8.6'.$

$$\tan \tfrac{1}{2} c = \frac{\sin \tfrac{1}{2}(B+A)}{\sin \tfrac{1}{2}(B-A)} \tan \tfrac{1}{2}(b-a).$$

$\log \sin \tfrac{1}{2}(B+A) = 9.6208$
$\log \csc \tfrac{1}{2}(B-A) = 0.8657$
$\log \tan \tfrac{1}{2}(b-a) = 9.4074$
$\log \tan \tfrac{1}{2} c = 9.8939$
$\tfrac{1}{2} c = 38° 4.2'$
$\therefore\ c = 76° 8.4'.$

Art. 169. — Page 127.

2. Here $s = 65° 30'$, $s - a = 27° 30'$, $s - b = 14° 30'$, $s - c = 23° 30'$.

$\log \sin (s-b) = 9.3986$
$\log \sin (s-c) = 9.6007$
$\log \csc s = 0.0410$
$\log \csc (s-a) = 0.3356$
$2)\overline{9.3759}$
$\log \tan \tfrac{1}{2} A = 9.6879$
$\tfrac{1}{2} A = 25° 59.1'$
$\therefore\ A = 51° 58.2'.$

$\log \sin (s-c) = 9.6007$
$\log \sin (s-a) = 9.6644$
$\log \csc s = 0.0410$
$\log \csc (s-b) = 0.6014$
$2)\overline{9.9075}$
$\log \tan \tfrac{1}{2} B = 9.9537$
$\tfrac{1}{2} B = 41° 57.2'$
$\therefore\ B = 83° 54.4'.$

$\log \sin (s-a) = 9.6644$
$\log \sin (s-b) = 9.3986$
$\log \csc s = 0.0410$
$\log \csc (s-c) = 0.3993$
$2)\overline{9.5033}$
$\log \tan \tfrac{1}{2} C = 9.7516$
$\tfrac{1}{2} C = 29° 26.6'$
$\therefore\ C = 58° 53.2'.$

3. Here $s = 105°$, $s - a = 4°$, $s - b = 56°$, $s - c = 45°$.

$\log \sin (s-b) = 9.9186$
$\log \sin (s-c) = 9.8495$
$\log \csc s = 0.0151$
$\log \csc (s-a) = 1.1564$
$2)\overline{0.9396}$
$\log \tan \tfrac{1}{2} A = 0.4698$
$\tfrac{1}{2} A = 71° 16.4'$
$\therefore\ A = 142° 32.8'.$

$\log \sin (s-c) = 9.8495$
$\log \sin (s-a) = 8.8436$
$\log \csc s = 0.0151$
$\log \csc (s-b) = 0.0814$
$2)\overline{8.7896}$
$\log \tan \tfrac{1}{2} B = 9.3948$
$\tfrac{1}{2} B = 13° 56.3'$
$\therefore\ B = 27° 52.6'.$

$$\begin{aligned}
\log \sin (s-a) &= 8.8436\\
\log \sin (s-b) &= 9.9186\\
\log \csc s &= 0.0151\\
\log \csc (s-c) &= 0.1505\\
&\underline{\qquad\qquad}\\
&2)\underline{8.9278}\\
\log \tan \tfrac{1}{2}C &= 9.4639\\
\tfrac{1}{2}C &= 16^\circ 13.6'\\
\therefore\ C &= 32^\circ 27.2'.
\end{aligned}$$

4. Here $s = 96^\circ$, $s-a = 35^\circ$, $s-b = 57^\circ$, $s-c = 4^\circ$.

$$\begin{aligned}
\log \sin (s-b) &= 9.9236\\
\log \sin (s-c) &= 8.8436\\
\log \csc s &= 0.0024\\
\log \csc (s-a) &= 0.2414\\
&\underline{\qquad\qquad}\\
&2)\underline{9.0110}\\
\log \tan \tfrac{1}{2}A &= 9.5055\\
\tfrac{1}{2}A &= 17^\circ 45.6'\\
\therefore\ A &= 35^\circ 31.2'.
\end{aligned}$$

$$\begin{aligned}
\log \sin (s-c) &= 8.8436\\
\log \sin (s-a) &= 9.7586\\
\log \csc s &= 0.0024\\
\log \csc (s-b) &= 0.0764\\
&\underline{\qquad\qquad}\\
&2)\underline{8.6810}\\
\log \tan \tfrac{1}{2}B &= 9.3405\\
\tfrac{1}{2}B &= 12^\circ 21.3'\\
\therefore\ B &= 24^\circ 42.6'.
\end{aligned}$$

$$\begin{aligned}
\log \sin (s-a) &= 9.7586\\
\log \sin (s-b) &= 9.9236\\
\log \csc s &= 0.0024\\
\log \csc (s-c) &= 1.1564\\
&\underline{\qquad\qquad}\\
&2)\underline{0.8410}\\
\log \tan \tfrac{1}{2}C &= 0.4205\\
\tfrac{1}{2}C &= 69^\circ 12.4'\\
\therefore\ C &= 138^\circ 24.8'.
\end{aligned}$$

5. Here $s = 107^\circ 10'$, $s-a = 44^\circ 50'$, $s-b = 53^\circ$, $s-c = 9^\circ 20'$.

$$\begin{aligned}
\log \sin (s-b) &= 9.9023\\
\log \sin (s-c) &= 9.2100\\
\log \csc s &= 0.0198\\
\log \csc (s-a) &= 0.1518\\
&\underline{\qquad\qquad}\\
&2)\underline{9.2839}\\
\log \tan \tfrac{1}{2}A &= 9.6419\\
\tfrac{1}{2}A &= 23^\circ 40.6'\\
\therefore\ A &= 47^\circ 21.2'.
\end{aligned}$$

$$\begin{aligned}
\log \sin (s-c) &= 9.2100\\
\log \sin (s-a) &= 9.8482\\
\log \csc s &= 0.0198\\
\log \csc (s-b) &= 0.0977\\
&\underline{\qquad\qquad}\\
&2)\underline{9.1757}\\
\log \tan \tfrac{1}{2}B &= 9.5878\\
\tfrac{1}{2}B &= 21^\circ 9.7'\\
\therefore\ B &= 42^\circ 19.4'.
\end{aligned}$$

$$\begin{aligned}
\log \sin (s-a) &= 9.8482 \\
\log \sin (s-b) &= 9.9023 \\
\log \csc s &= 0.0198 \\
\log \csc (s-c) &= 0.7900 \\
& \underline{\quad\quad} \\
& 2)\underline{0.5603} \\
\log \tan \tfrac{1}{2}C &= 0.2801 \\
\tfrac{1}{2}C &= 62^\circ 19' \\
\therefore\ C &= 124^\circ 38'.
\end{aligned}$$

Art. 170. — Page 128.

2. Here $S = 109^\circ$, $S-A = 34^\circ$, $S-B = 27^\circ$, $S-C = 48^\circ$.

$$\begin{aligned}
\log \cos S &= 9.5126 \\
\log \cos (S-A) &= 9.9186 \\
\log \sec (S-B) &= 0.0501 \\
\log \sec (S-C) &= 0.1745 \\
& \underline{\quad\quad} \\
& 2)\underline{9.6558} \\
\log \tan \tfrac{1}{2}a &= 9.8279 \\
\tfrac{1}{2}a &= 33^\circ 55.9' \\
\therefore\ a &= 67^\circ 51.8'.
\end{aligned}$$

$$\begin{aligned}
\log \cos S &= 9.5126 \\
\log \cos (S-B) &= 9.9499 \\
\log \sec (S-C) &= 0.1745 \\
\log \sec (S-A) &= 0.0814 \\
& \underline{\quad\quad} \\
& 2)\underline{9.7184} \\
\log \tan \tfrac{1}{2}b &= 9.8592 \\
\tfrac{1}{2}b &= 35^\circ 52.2' \\
\therefore\ b &= 71^\circ 44.4'.
\end{aligned}$$

$$\begin{aligned}
\log \cos S &= 9.5126 \\
\log \cos (S-C) &= 9.8255 \\
\log \sec (S-A) &= 0.0814 \\
\log \sec (S-B) &= 0.0501 \\
& \underline{\quad\quad} \\
& 2)\underline{9.4696} \\
\log \tan \tfrac{1}{2}c &= 9.7348 \\
\tfrac{1}{2}c &= 28^\circ 30' \\
\therefore\ c &= 57^\circ.
\end{aligned}$$

3. Here $S = 165^\circ$, $S-A = 45^\circ$, $S-B = 35^\circ$, $S-C = 85^\circ$.

$$\begin{aligned}
\log \cos S &= 9.9849 \\
\log \cos (S-A) &= 9.8495 \\
\log \sec (S-B) &= 0.0866 \\
\log \sec (S-C) &= 1.0597 \\
& \underline{\quad\quad} \\
& 2)\underline{0.9807} \\
\log \tan \tfrac{1}{2}a &= 0.4903 \\
\tfrac{1}{2}a &= 72^\circ 4.9' \\
\therefore\ a &= 144^\circ 9.8'.
\end{aligned}$$

$$\begin{aligned}
\log \cos S &= 9.9849 \\
\log \cos (S-B) &= 9.9134 \\
\log \sec (S-C) &= 1.0597 \\
\log \sec (S-A) &= 0.1505 \\
& \underline{\quad\quad} \\
& 2)\underline{1.1085} \\
\log \tan \tfrac{1}{2}b &= 0.5542 \\
\tfrac{1}{2}b &= 74^\circ 24.3' \\
\therefore\ b &= 148^\circ 48.6'.
\end{aligned}$$

$\log \cos S = 9.9849$
$\log \cos (S - C) = 8.9403$
$\log \sec (S - A) = 0.1505$
$\log \sec (S - B) = 0.0866$

$2)\underline{9.1623}$
$\log \tan \frac{1}{2} c = 9.5811$
$\frac{1}{2} c = 20^\circ 51.8'$
$\therefore\ c = 41^\circ 43.6'.$

4. Here $S = 124^\circ 40'$, $S - A = 33^\circ 30'$, $S - B = 39^\circ$, $S - C = 52^\circ 10'$.

$\log \cos S = 9.7550$
$\log \cos (S - A) = 9.9211$
$\log \sec (S - B) = 0.1095$
$\log \sec (S - C) = 0.2123$

$2)\underline{9.9979}$
$\log \tan \frac{1}{2} a = 9.9989$
$\frac{1}{2} a = 44^\circ 55.6'$
$\therefore\ a = 89^\circ 51.2'.$

$\log \cos S = 9.7550$
$\log \cos (S - B) = 9.8905$
$\log \sec (S - C) = 0.2123$
$\log \sec (S - A) = 0.0789$

$2)\underline{9.9367}$
$\log \tan \frac{1}{2} b = 9.9683$
$\frac{1}{2} b = 42^\circ 54.6'$
$\therefore\ b = 85^\circ 49.2'.$

$\log \cos S = 9.7550$
$\log \cos (S - C) = 9.7877$
$\log \sec (S - A) = 0.0789$
$\log \sec (S - B) = 0.1095$

$2)\underline{9.7311}$
$\log \tan \frac{1}{2} c = 9.8655$
$\frac{1}{2} c = 36^\circ 15.9'$
$\therefore\ c = 72^\circ 31.8'.$

5. Here
$S = 102^\circ 40'$, $S - A = -35^\circ 36'$, $S - B = 71^\circ 29'$, $S - C = 66^\circ 47'$.

$\log \cos S = 9.3410$
$\log \cos (S - A) = 9.9102$
$\log \sec (S - B) = 0.4981$
$\log \sec (S - C) = 0.4043$

$2)\underline{0.1536}$
$\log \tan \frac{1}{2} a = 0.0768$
$\frac{1}{2} a = 50^\circ 2.3'$
$\therefore\ a = 100^\circ 4.6'.$

$\log \cos S = 9.3410$
$\log \cos (S - B) = 9.5019$
$\log \sec (S - C) = 0.4043$
$\log \sec (S - A) = 0.0898$

$2)\underline{9.3370}$
$\log \tan \frac{1}{2} b = 9.6685$
$\frac{1}{2} b = 24^\circ 59.4'$
$\therefore\ b = 49^\circ 58.8'.$

$$\begin{aligned}
\log\cos S &= 9.3410\\
\log\cos(S-C) &= 9.5957\\
\log\sec(S-A) &= 0.0898\\
\log\sec(S-B) &= 0.4981\\
&\quad 2)\overline{9.5246}\\
\log\tan\tfrac{1}{2}c &= 9.7623\\
\tfrac{1}{2}c &= 30^\circ\,3'\\
\therefore\ c &= 60^\circ\,6'.
\end{aligned}$$

Art. 171. — Page 131.

4. $\sin C = \dfrac{\sin c \sin B}{\sin b}$.

$$\begin{aligned}
\log\sin c &= 9.9549\\
\log\csc b &= 0.0062\\
\log\sin B &= 9.9979\\
\log\sin C &= 9.9590\\
\therefore\ C &= 65^\circ\,30'.
\end{aligned}$$

$$\begin{aligned}
\tfrac{1}{2}(b+c) &= 82^\circ,\\
\tfrac{1}{2}(b-c) &= 17^\circ\,40'.
\end{aligned}$$

$$\begin{aligned}
\tfrac{1}{2}(B+C) &= 80^\circ\,35',\\
\tfrac{1}{2}(B-C) &= 15^\circ\ \ 5'.
\end{aligned}$$

$\cot\frac{1}{2}A = \dfrac{\sin\frac{1}{2}(b+c)}{\sin\frac{1}{2}(b-c)}\tan\frac{1}{2}(B-C)$.

$$\begin{aligned}
\log\sin\tfrac{1}{2}(b+c) &= 9.9958\\
\log\csc\tfrac{1}{2}(b-c) &= 0.5179\\
\log\tan\tfrac{1}{2}(B-C) &= 9.4306\\
\log\cot\tfrac{1}{2}A &= 9.9443\\
\tfrac{1}{2}A &= 48^\circ\,40'\\
\therefore\ A &= 97^\circ\,20'.
\end{aligned}$$

$\tan\frac{1}{2}a = \dfrac{\sin\frac{1}{2}(B+C)}{\sin\frac{1}{2}(B-C)}\tan\frac{1}{2}(b-c)$.

$$\begin{aligned}
\log\sin\tfrac{1}{2}(B+C) &= 9.9941\\
\log\csc\tfrac{1}{2}(B-C) &= 0.5847\\
\log\tan\tfrac{1}{2}(b-c) &= 9.5031\\
\log\tan\tfrac{1}{2}a &= 0.0819\\
\tfrac{1}{2}a &= 50^\circ\,22.3'\\
\therefore\ a &= 100^\circ\,44.6'.
\end{aligned}$$

5. $\sin B = \dfrac{\sin b \sin A}{\sin a}$.

$$\begin{aligned}
\log\sin b &= 9.9446\\
\log\csc a &= 0.1919\\
\log\sin A &= 9.6946\\
\log\sin B &= 9.8311\\
\therefore\ B &= 42^\circ\,40',\ \text{or}\ 137^\circ\,20'.
\end{aligned}$$

$$\begin{aligned}
\tfrac{1}{2}(b+a) &= 79^\circ\,10',\\
\tfrac{1}{2}(b-a) &= 39^\circ\,10'.
\end{aligned}$$

$$\begin{aligned}
\tfrac{1}{2}(B+A) &= 36^\circ\,10',\ \text{or}\ 83^\circ\,30'.\\
\tfrac{1}{2}(B-A) &= \ 6^\circ\,30',\ \text{or}\ 53^\circ\,50'.
\end{aligned}$$

$\cot\frac{1}{2}C = \dfrac{\sin\frac{1}{2}(b+a)}{\sin\frac{1}{2}(b-a)}\tan\frac{1}{2}(B-A)$.

$\tan\frac{1}{2}c = \dfrac{\sin\frac{1}{2}(B+A)}{\sin\frac{1}{2}(B-A)}\tan\frac{1}{2}(b-a)$.

Using the first value of B, we have:

$$\begin{aligned}
\log\sin\tfrac{1}{2}(b+a) &= 9.9922\\
\log\csc\tfrac{1}{2}(b-a) &= 0.1996\\
\log\tan\tfrac{1}{2}(B-A) &= 9.0567\\
\log\cot\tfrac{1}{2}C &= 9.2485\\
\tfrac{1}{2}C &= \ 79^\circ\,57'\\
\therefore\ C &= 159^\circ\,54'.
\end{aligned}$$

$$\begin{aligned}
\log\sin\tfrac{1}{2}(B+A) &= 9.7710\\
\log\csc\tfrac{1}{2}(B-A) &= 0.9461\\
\log\tan\tfrac{1}{2}(b-a) &= 9.9110\\
\log\tan\tfrac{1}{2}c &= 0.6281\\
\tfrac{1}{2}c &= 76^\circ\,45.1'\\
\therefore\ c &= 153^\circ\,30.2'.
\end{aligned}$$

Using the second value of B, we have:

$\log \sin \frac{1}{2}(b+a) = 9.9922$
$\log \csc \frac{1}{2}(b-a) = 0.1996$
$\log \tan \frac{1}{2}(B-A) = 0.1361$
$\log \cot \frac{1}{2}C = 0.3279$
$\frac{1}{2}C = 25^\circ 10.3'$
$\therefore\ C = 50^\circ 20.6'.$

$\log \sin \frac{1}{2}(B+A) = 9.9972$
$\log \csc \frac{1}{2}(B-A) = 0.0930$
$\log \tan \frac{1}{2}(b-a) = 9.9110$
$\log \tan \frac{1}{2}c = 0.0012$
$\frac{1}{2}c = 45^\circ 4.8'$
$\therefore\ c = 90^\circ 9.6'.$

6. $$\sin A = \frac{\sin a \sin C}{\sin c}.$$

$\log \sin a = 9.9561$
$\log \csc c = 0.2562$
$\log \sin C = 9.7973$
$\log \sin A = 0.0096$

Since $\log \sin A$ is *positive*, the triangle is impossible.

7. $$\sin C = \frac{\sin c \sin A}{\sin a}.$$

$\log \sin c = 9.9958$
$\log \csc a = 0.0252$
$\log \sin A = 9.9790$
$\log \sin C = 0.0000$
$\therefore\ C = 90^\circ.$

$$\overset{-}{\cos B} = \frac{\overset{-}{\tan a}}{\overset{+}{\tan c}}.$$

$\log \tan a = 0.4549$
$\log \tan c = 0.8522$
$\log \cos B = 9.6027$
$180^\circ - B = 66^\circ 23.1'$
$\therefore\ B = 113^\circ 36.9'.$

$$\overset{-}{\cos b} = \frac{\overset{+}{\cos c}}{\overset{-}{\cos a}}.$$

$\log \cos c = 9.1436$
$\log \cos a = 9.5199$
$\log \cos b = 9.6237$
$180^\circ - b = 65^\circ 8.1'$
$\therefore\ b = 114^\circ 51.9'.$

8. $$\sin B = \frac{\sin b \sin C}{\sin c}.$$

$\log \sin b = 9.9770$
$\log \csc c = 0.1845$
$\log \sin C = 9.8066$
$\log \sin B = 9.9681$
$\therefore\ B = 68^\circ 18'$, or $111^\circ 42'$.

$\frac{1}{2}(b+c) = 74^\circ 40'$,
$\frac{1}{2}(b-c) = 33^\circ 50'$.

$\frac{1}{2}(B+C) = 54^\circ 4'$, or $75^\circ 46'$.
$\frac{1}{2}(B-C) = 14^\circ 14'$, or $35^\circ 56'$.

$$\cot \tfrac{1}{2}A = \frac{\sin \frac{1}{2}(b+c)}{\sin \frac{1}{2}(b-c)} \tan \tfrac{1}{2}(B-C). \qquad \tan \tfrac{1}{2}a = \frac{\sin \frac{1}{2}(B+C)}{\sin \frac{1}{2}(B-C)} \tan \tfrac{1}{2}(b-c).$$

Using the first value of B, we have:

$\log \sin \frac{1}{2}(b+c) = 9.9843$
$\log \csc \frac{1}{2}(b-c) = 0.2543$
$\log \tan \frac{1}{2}(B-C) = 9.4042$
$\log \cot \frac{1}{2}A = 9.6428$
$\frac{1}{2}A = 66^\circ 16.9'$
$\therefore A = 132^\circ 33.8'.$

$\log \sin \frac{1}{2}(B+C) = 9.9084$
$\log \csc \frac{1}{2}(B-C) = 0.6093$
$\log \tan \frac{1}{2}(b-c) = 9.8263$
$\log \tan \frac{1}{2}a = 0.3440$
$\frac{1}{2}a = 65^\circ 37.9'$
$\therefore a = 131^\circ 15.8'.$

Using the second value of B, we have:

$\log \sin \frac{1}{2}(b+c) = 9.9843$
$\log \csc \frac{1}{2}(b-c) = 0.2543$
$\log \tan \frac{1}{2}(B-C) = 9.8602$
$\log \cot \frac{1}{2}A = 0.0988$
$\frac{1}{2}A = 38^\circ 32.3'$
$\therefore A = 77^\circ 4.6'.$

$\log \sin \frac{1}{2}(B+C) = 9.9865$
$\log \csc \frac{1}{2}(B-C) = 0.2315$
$\log \tan \frac{1}{2}(b-c) = 9.8263$
$\log \tan \frac{1}{2}a = 0.0443$
$\frac{1}{2}a = 47^\circ 55'$
$\therefore a = 95^\circ 50'.$

9. $\sin A = \dfrac{\sin a \sin B}{\sin b}.$

$\log \sin a = 9.4821$
$\log \csc b = 0.5686$
$\log \sin B = 9.9134$
$\log \sin A = 9.9641$
$\therefore A = 67^\circ 1.7'$, or $112^\circ 58.3'$.

Since both values of A are $< B$, while a is given $> b$, the triangle is impossible.

10. $\sin C = \dfrac{\sin c \sin A}{\sin a}.$

$\log \sin c = 9.8241$
$\log \csc a = 0.0866$
$\log \sin A = 9.8297$
$\log \sin C = 9.7404$
$\therefore C = 146^\circ 37.9'.$

$\frac{1}{2}(c+a) = 96^\circ 35',$
$\frac{1}{2}(c-a) = 41^\circ 35'.$

$\frac{1}{2}(C+A) = 94^\circ 33.95',$
$\frac{1}{2}(C-A) = 52^\circ\ 3.95'.$

$\cot \frac{1}{2}B = \dfrac{\sin \frac{1}{2}(c+a)}{\sin \frac{1}{2}(c-a)} \tan \frac{1}{2}(C-A).$

$\log \sin \frac{1}{2}(c+a) = 9.9972$
$\log \csc \frac{1}{2}(c-a) = 0.1780$
$\log \tan \frac{1}{2}(C-A) = 0.1082$
$\log \cot \frac{1}{2}B = 0.2834$
$\frac{1}{2}B = 27^\circ 30.3'$
$\therefore B = 55^\circ 0.6'.$

$\tan \frac{1}{2}b = \dfrac{\sin \frac{1}{2}(C+A)}{\sin \frac{1}{2}(C-A)} \tan \frac{1}{2}(c-a).$

$\log \sin \frac{1}{2}(C+A) = 9.9987$
$\log \csc \frac{1}{2}(C-A) = 0.1031$
$\log \tan \frac{1}{2}(c-a) = 9.9481$
$\log \tan \frac{1}{2}b = 0.0499$
$\frac{1}{2}b = 48^\circ 17.2'$
$\therefore b = 96^\circ 34.4'.$

Art. 172. — Page 133.

2. $\sin b = \dfrac{\sin B \sin c}{\sin C}$.

$\log \sin B = 9.9537$
$\log \csc C = 0.0066$
$\log \sin c = 9.9976$
$\log \sin b = 9.9579$
$\therefore\ b = 114° 50'.$

$\frac{1}{2}(B + C) = 98°,$
$\frac{1}{2}(B - C) = 18°.$

$\frac{1}{2}(b + c) = 99° 25',$
$\frac{1}{2}(b - c) = 15° 25'.$

$\cot \frac{1}{2} A = \dfrac{\sin \frac{1}{2}(b+c)}{\sin \frac{1}{2}(b-c)} \tan \frac{1}{2}(B-C).$ $\tan \frac{1}{2} a = \dfrac{\sin \frac{1}{2}(B+C)}{\sin \frac{1}{2}(B-C)} \tan \frac{1}{2}(b-c).$

$\log \sin \frac{1}{2}(b + c) = 9.9941$
$\log \csc \frac{1}{2}(b - c) = 0.5754$
$\log \tan \frac{1}{2}(B - C) = 9.5118$
$\log \cot \frac{1}{2} A = 0.0813$
$\frac{1}{2} A = 39° 40'$
$\therefore\ A = 79° 20'.$

$\log \sin \frac{1}{2}(B + C) = 9.9958$
$\log \csc \frac{1}{2}(B - C) = 0.5100$
$\log \tan \frac{1}{2}(b - c) = 9.4405$
$\log \tan \frac{1}{2} a = 9.9463$
$\frac{1}{2} a = 41° 28'$
$\therefore\ a = 82° 56'.$

3. $\sin a = \dfrac{\sin A \sin b}{\sin B}$.

$\log \sin A = 9.8711$
$\log \csc B = 0.1919$
$\log \sin b = 9.9023$
$\log \sin a = 9.9653$
$\therefore\ a = 67° 24'$, or $112° 36'$.

$\frac{1}{2}(B + A) = 136°,$
$\frac{1}{2}(B - A) = 4°.$

$\frac{1}{2}(b + a) = 97° 12'$, or $119° 48'$.
$\frac{1}{2}(b - a) = 29° 48'$, or $7° 12'$.

$\cot \frac{1}{2} C = \dfrac{\sin \frac{1}{2}(b+a)}{\sin \frac{1}{2}(b-a)} \tan \frac{1}{2}(B-A).$ $\tan \frac{1}{2} c = \dfrac{\sin \frac{1}{2}(B+A)}{\sin \frac{1}{2}(B-A)} \tan \frac{1}{2}(b-a).$

Using the first value of a, we have:

$\log \sin \frac{1}{2}(b + a) = 9.9966$
$\log \csc \frac{1}{2}(b - a) = 0.3036$
$\log \tan \frac{1}{2}(B - A) = 8.8446$
$\log \cot \frac{1}{2} C = 9.1448$
$\frac{1}{2} C = 82° 3.2'$
$\therefore\ C = 164° 6.4'.$

$\log \sin \frac{1}{2}(B + A) = 9.8418$
$\log \csc \frac{1}{2}(B - A) = 1.1564$
$\log \tan \frac{1}{2}(b - a) = 9.7579$
$\log \tan \frac{1}{2} c = 0.7561$
$\frac{1}{2} c = 80° 3.2$
$\therefore\ c = 160° 6.4'.$

Using the second value of a, we have:

$\log \sin \frac{1}{2}(b + a) = 9.9384$
$\log \csc \frac{1}{2}(b - a) = 0.9019$
$\log \tan \frac{1}{2}(B - A) = 8.8446$
$\log \cot \frac{1}{2} C = 9.6849$
$\frac{1}{2} C = 64° 10.3'$
$\therefore\ C = 128° 20.6'.$

$\log \sin \frac{1}{2}(B + A) = 9.8418$
$\log \csc \frac{1}{2}(B - A) = 1.1564$
$\log \tan \frac{1}{2}(b - a) = 9.1015$
$\log \tan \frac{1}{2} c = 0.0997$
$\frac{1}{2} c = 51° 31.2'$
$\therefore\ c = 103° 2.4'.$

4. $$\sin c = \frac{\sin C \sin a}{\sin A}.$$

$$\begin{aligned} \log \sin C &= 9.9904 \\ \log \csc A &= 0.0541 \\ \log \sin a &= 9.9555 \\ \log \sin c &= \overline{0.0000} \end{aligned}$$

$$\therefore c = 90^\circ.$$

The triangle is a quadrantal triangle, and the sides a' and c' of its polar triangle are 118° and 78°.

$$\cos b' = \frac{\cos c'}{\cos a'}. \qquad \cos B' = \frac{\tan a'}{\tan c'}.$$

$$\begin{aligned} \log \cos c' &= 9.3179 & \log \tan a' &= 0.2743 \\ \log \cos a' &= 9.6716 & \log \tan c' &= 0.6725 \\ \log \cos b' &= 9.6463 & \log \cos B' &= 9.6018 \\ \therefore 180^\circ - b' &= 63^\circ 42.7'. & \therefore 180^\circ - B' &= 66^\circ 26.2'. \end{aligned}$$

Therefore in the given triangle, $B = 63^\circ 42.7'$, and $b = 66^\circ 26.2'$.

5. $$\sin b = \frac{\sin B \sin a}{\sin A}.$$

$$\begin{aligned} \log \sin B &= 9.9624 \\ \log \csc A &= 0.1418 \\ \log \sin a &= 9.9948 \\ \log \sin b &= \overline{0.0990} \end{aligned}$$

Since $\log \sin b$ is positive, the triangle is impossible.

6. $$\sin b = \frac{\sin B \sin c}{\sin C}.$$

$$\begin{aligned} \log \sin B &= 9.5798 \\ \log \csc C &= 0.2600 \\ \log \sin c &= 9.8227 \\ \log \sin b &= \overline{9.6625} \\ \therefore b &= 27^\circ 22.1'. \end{aligned}$$

$$\tfrac{1}{2}(C+B) = 84^\circ 30', \quad \tfrac{1}{2}(C-B) = 62^\circ 10'.$$

$$\tfrac{1}{2}(c+b) = 82^\circ 51.05', \quad \tfrac{1}{2}(c-b) = 55^\circ 28.95'.$$

$$\cot \tfrac{1}{2}A = \frac{\sin \frac{1}{2}(c+b)}{\sin \frac{1}{2}(c-b)} \tan \tfrac{1}{2}(C-B). \qquad \tan \tfrac{1}{2}a = \frac{\sin \frac{1}{2}(C+B)}{\sin \frac{1}{2}(C-B)} \tan \tfrac{1}{2}(c-b).$$

$$\begin{aligned} \log \sin \tfrac{1}{2}(c+b) &= 9.9966 & \log \sin \tfrac{1}{2}(C+B) &= 9.9980 \\ \log \csc \tfrac{1}{2}(c-b) &= 0.0841 & \log \csc \tfrac{1}{2}(C-B) &= 0.0534 \\ \log \tan \tfrac{1}{2}(C-B) &= 0.2774 & \log \tan \tfrac{1}{2}(c-b) &= 0.1626 \\ \log \cot \tfrac{1}{2}A &= 0.3581 & \log \tan \tfrac{1}{2}a &= 0.2140 \\ \tfrac{1}{2}A &= 23^\circ 40.6' & \tfrac{1}{2}a &= 58^\circ 34.6' \\ \therefore A &= 47^\circ 21.2'. & \therefore a &= 117^\circ 9.2'. \end{aligned}$$

7. $\sin a = \dfrac{\sin A \sin c}{\sin C}$.

$\log \sin A = 9.9446$
$\log \csc C = 0.1950$
$\log \sin c = 9.6946$
$\log \sin a = 9.8342$
$\therefore\ a = 43^\circ 3.1'$, or $136^\circ 56.9'$.

$\frac{1}{2}(C+A) = 101^\circ$,
$\frac{1}{2}(C-A) = 39^\circ 20'$.
$\frac{1}{2}(c+a) = 96^\circ 41.55'$,
or $143^\circ 38.45'$.
$\frac{1}{2}(c-a) = 53^\circ 38.45'$,
or $6^\circ 41.55'$.

$\cot \frac{1}{2}B = \dfrac{\sin \frac{1}{2}(c+a)}{\sin \frac{1}{2}(c-a)} \tan \frac{1}{2}(C-A)$. $\quad \tan \frac{1}{2}b = \dfrac{\sin \frac{1}{2}(C+A)}{\sin \frac{1}{2}(C-A)} \tan \frac{1}{2}(c-a)$.

Using the first value of a, we have:

$\log \sin \frac{1}{2}(c+a) = 9.9971$
$\log \csc \frac{1}{2}(c-a) = 0.0940$
$\log \tan \frac{1}{2}(C-A) = 9.9135$
$\log \cot \frac{1}{2}B = 0.0046$
$\frac{1}{2}B = 44^\circ 41.9'$
$\therefore\ B = 89^\circ 23.8'$.

$\log \sin \frac{1}{2}(C+A) = 9.9919$
$\log \csc \frac{1}{2}(C-A) = 0.1980$
$\log \tan \frac{1}{2}(c-a) = 0.1330$
$\log \tan \frac{1}{2}b = 0.3229$
$\frac{1}{2}b = 64^\circ 34.2'$
$\therefore\ b = 129^\circ\ 8.4'$.

Using the second value of a, we have:

$\log \sin \frac{1}{2}(c+a) = 9.7730$
$\log \csc \frac{1}{2}(c-a) = 0.9335$
$\log \tan \frac{1}{2}(C-A) = 9.9135$
$\log \cot \frac{1}{2}B = 0.6200$
$\frac{1}{2}B = 13^\circ 29.3'$
$\therefore\ B = 26^\circ 58.6'$.

$\log \sin \frac{1}{2}(C+A) = 9.9919$
$\log \csc \frac{1}{2}(C-A) = 0.1980$
$\log \tan \frac{1}{2}(c-a) = 9.0695$
$\log \tan \frac{1}{2}b = 9.2594$
$\frac{1}{2}b = 10^\circ 17.9'$
$\therefore\ b = 20^\circ 35.8'$.

8. $\sin c = \dfrac{\sin C \sin b}{\sin B}$.

$\log \sin C = 9.9950$
$\log \csc B = 0.0194$
$\log \sin b = 9.9252$
$\log \sin c = 9.9396$
$\therefore\ c = 60^\circ 28.6'$, or $119^\circ 31.4'$.

Since both values of c are $< b$, while C is given $> B$, the triangle is impossible.

Art. 173. — Pages 134 and 136.

1. Let B and G denote the positions of Boston and Greenwich, respectively, and P the north-pole.

Denote the arcs PG, PB, and BG by b, g, and p.

Then in the triangle PBG,

$\angle P = 71^\circ 4'$, $b = 90^\circ - 51^\circ 29' = 38^\circ 31'$, $g = 90^\circ - 42^\circ 21' = 47^\circ 39'$.

By Art. 162, we have:

$$\tan \tfrac{1}{2}(G - B) = \frac{\sin \frac{1}{2}(g - b)}{\sin \frac{1}{2}(g + b)} \cot \tfrac{1}{2} P,$$

$$\tan \tfrac{1}{2}(G + B) = \frac{\cos \frac{1}{2}(g - b)}{\cos \frac{1}{2}(g + b)} \cot \tfrac{1}{2} P.$$

From the data,

$$\tfrac{1}{2}(g - b) = 4^\circ 34', \quad \tfrac{1}{2}(g + b) = 43^\circ 5', \quad \tfrac{1}{2} P = 35^\circ 32'.$$

$\log \sin \frac{1}{2}(g - b)$	$= 8.9009$	$\log \cos \frac{1}{2}(g - b)$	$= 9.9987$
$\log \csc \frac{1}{2}(g + b)$	$= 0.1656$	$\log \sec \frac{1}{2}(g + b)$	$= 0.1365$
$\log \cot \frac{1}{2} P$	$= 0.1462$	$\log \cot \frac{1}{2} P$	$= 0.1462$
$\log \tan \frac{1}{2}(G - B)$	$= 9.2127$	$\log \tan \frac{1}{2}(G + B)$	$= 0.2814$
$\therefore \frac{1}{2}(G - B)$	$= 9^\circ 16.1'$.	$\therefore \frac{1}{2}(G + B)$	$= 62^\circ 23.2'$.

$$\therefore B = \tfrac{1}{2}(G + B) - \tfrac{1}{2}(G - B) = 53^\circ 7.1',$$

and

$$G = \tfrac{1}{2}(G + B) + \tfrac{1}{2}(G - B) = 71^\circ 39.3'.$$

$$\tan \tfrac{1}{2} p = \frac{\sin \frac{1}{2}(G + B)}{\sin \frac{1}{2}(G - B)} \tan \tfrac{1}{2}(g - b).$$

$\log \sin \frac{1}{2}(G + B)$	$= 9.9475$
$\log \csc \frac{1}{2}(G - B)$	$= 0.7930$
$\log \tan \frac{1}{2}(g - b)$	$= 8.9023$
$\log \tan \frac{1}{2} p$	$= 9.6428$
$\frac{1}{2} p$	$= 23^\circ 43.1'$
$\therefore p$	$= 47^\circ 26.2'$.

$$47^\circ 26.2' = 2846.2'.$$

$$360^\circ = 21600'.$$

Circumference of earth $= 7912 \times 3.1416$.

$$\therefore p \text{ (in miles)} = \frac{2846.2}{21600} \times 7912 \times 3.1416.$$

$\log 2846.2$	$= 3.4542$
$\operatorname{colog} 21600$	$= 5.6655$
$\log 7912$	$= 3.8983$
$\log 3.1416$	$= 0.4971$
$\log p$	$= 3.5151$
$\therefore p$	$= 3274.3$.

Therefore the shortest distance between the places is 3274.3 miles; the bearing of Greenwich from Boston is N. 53° 7.1′ E., and of Boston from Greenwich, N. 71° 39.3′ W.

2. Let C and V denote the positions of Calcutta and Valparaiso, respectively, and P the north-pole.

Denote the arcs PV, PC, and CV by c, v, and p.

Then in the triangle PCV,

$$\angle P = 88^\circ 19' + 71^\circ 42' = 160^\circ 1',\quad c = 90^\circ + 33^\circ 2' = 123^\circ 2',$$
$$v = 90^\circ - 22^\circ 33' = 67^\circ 27'.$$

By Art. 162, we have:

$$\tan \tfrac{1}{2}(C - V) = \frac{\sin \frac{1}{2}(c - v)}{\sin \frac{1}{2}(c + v)} \cot \tfrac{1}{2} P,$$

$$\tan \tfrac{1}{2}(C + V) = \frac{\cos \frac{1}{2}(c - v)}{\cos \frac{1}{2}(c + v)} \cot \tfrac{1}{2} P.$$

From the data, $\frac{1}{2}(c-v) = 27^\circ 47.5'$, $\frac{1}{2}(c+v) = 95^\circ 14.5'$, $\frac{1}{2} P = 80^\circ 0.5'$.

$\log \sin \frac{1}{2}(c-v)$	$= 9.6686$	$\log \cos \frac{1}{2}(c-v)$	$= 9.9468$
$\log \csc \frac{1}{2}(c+v)$	$= 0.0018$	$\log \sec \frac{1}{2}(c+v)$	$= 1.0393$
$\log \cot \frac{1}{2} P$	$= 9.2459$	$\log \cot \frac{1}{2} P$	$= 9.2459$
$\log \tan \frac{1}{2}(C-V)$	$= 8.9163$	$\log \tan \frac{1}{2}(C+V)$	$= 0.2320$
$\therefore \frac{1}{2}(C-V)$	$= 4^\circ 42.9'$.	$180^\circ - \frac{1}{2}(C+V)$	$= 59^\circ 37.5'$
		$\therefore \frac{1}{2}(C+V)$	$= 120^\circ 22.5'$.

$$\therefore C = \tfrac{1}{2}(C+V) + \tfrac{1}{2}(C-V) = 125^\circ 5.4',$$

and

$$V = \tfrac{1}{2}(C+V) - \tfrac{1}{2}(C-V) = 115^\circ 39.6'.$$

$$\tan \tfrac{1}{2} p = \frac{\sin \frac{1}{2}(C+V)}{\sin \frac{1}{2}(C-V)} \tan \tfrac{1}{2}(c-v).$$

$$\log \sin \tfrac{1}{2}(C+V) = 9.9359$$
$$\log \csc \tfrac{1}{2}(C-V) = 1.0852$$
$$\log \tan \tfrac{1}{2}(c-v) = 9.7218$$
$$\log \tan \tfrac{1}{2} p = 0.7429$$
$$\tfrac{1}{2} p = 79^\circ 45.2'$$
$$\therefore p = 159^\circ 30.4'.$$

$$159^\circ 30.4' = 9570.4'.$$
$$360^\circ = 21600'.$$

Circumference of earth $= 7912 \times 3.1416$.

$$\therefore p \text{ (in miles)} = \frac{9570.4}{21600} \times 7912 \times 3.1416.$$

$$\log 9570.4 = 3.9809$$
$$\operatorname{colog} 21600 = 5.6655$$
$$\log 7912 = 3.8983$$
$$\log 3.1416 = 0.4971$$
$$\log p = 4.0418$$
$$\therefore p = 11010.$$

Therefore the shortest distance between the places is 11010 miles; the bearing of Calcutta from Valparaiso is N. 115° 39.6′ E., or S. 64° 20.4′ E., and of Valparaiso from Calcutta is N. 125° 5.4′ W., or S. 54° 54.6′ W.

3. Let S and Q denote the positions of Sandy Hook and Queenstown, respectively, P the north pole, and X the intersection of the arc QS with the meridian of 50° W.

Denote the arcs PQ and PS by s and q.

Then in the triangle PQS,

$$\angle P = 74° 1′ - 8° 19′ = 65° 42′, \quad q = 90° - 40° 28′ = 49° 32′,$$
$$s = 90° - 51° 50′ = 38° 10′.$$

By Art. 162, we have:

$$\tan \tfrac{1}{2}(Q - S) = \frac{\sin \frac{1}{2}(q - s)}{\sin \frac{1}{2}(q + s)} \cot \tfrac{1}{2} P,$$

$$\tan \tfrac{1}{2}(Q + S) = \frac{\cos \frac{1}{2}(q - s)}{\cos \frac{1}{2}(q + s)} \cot \tfrac{1}{2} P.$$

From the data,

$$\tfrac{1}{2}(q - s) = 5° 41′, \ \tfrac{1}{2}(q + s) = 43° 51′, \ \tfrac{1}{2} P = 32° 51′.$$

$\log \sin \frac{1}{2}(q - s)$	$= 8.9957$	$\log \cos \frac{1}{2}(q - s)$	$= 9.9979$
$\log \csc \frac{1}{2}(q + s)$	$= 0.1594$	$\log \sec \frac{1}{2}(q + s)$	$= 0.1419$
$\log \cot \frac{1}{2} P$	$= 0.1900$	$\log \cot \frac{1}{2} P$	$= 0.1900$
$\log \tan \frac{1}{2}(Q - S)$	$= 9.3451$	$\log \tan \frac{1}{2}(Q + S)$	$= 0.3298$
$\therefore \frac{1}{2}(Q - S)$	$= 12° 28.9′.$	$\therefore \frac{1}{2}(Q + S)$	$= 64° 55.5′.$

Then in the triangle PQX,

$$\angle P = 50° - 8° 19′ = 41° 41′,$$
$$\angle Q = \tfrac{1}{2}(Q + S) + \tfrac{1}{2}(Q - S) = 77° 24.4′, \ PQ = 38° 10′.$$

By Arts. 160 and 161, we have:

$$\tan \tfrac{1}{2}(PX - QX) = \frac{\sin \frac{1}{2}(Q - QPX)}{\sin \frac{1}{2}(Q + QPX)} \tan \tfrac{1}{2} PQ.$$

$$\tan \tfrac{1}{2}(PX + QX) = \frac{\cos \frac{1}{2}(Q - QPX)}{\cos \frac{1}{2}(Q + QPX)} \tan \tfrac{1}{2} PQ.$$

From the data,

$$\tfrac{1}{2}(Q - QPX) = 17° 51.7′, \ \tfrac{1}{2}(Q + QPX) = 59° 32.7′, \ \tfrac{1}{2} PQ = 19° 5′.$$

$\log \sin \frac{1}{2}(Q - QPX)$	$= 9.4868$	$\log \cos \frac{1}{2}(Q - QPX)$	$= 9.9785$
$\log \csc \frac{1}{2}(Q + QPX)$	$= 0.0645$	$\log \sec \frac{1}{2}(Q + QPX)$	$= 0.2951$
$\log \tan \frac{1}{2} PQ$	$= 9.5390$	$\log \tan \frac{1}{2} PQ$	$= 9.5390$
$\log \tan \frac{1}{2}(PX - QX)$	$= 9.0903$	$\log \tan \frac{1}{2}(PX + QX)$	$= 9.8126$
$\therefore \frac{1}{2}(PX - QX)$	$= 7° 1.2′.$	$\therefore \frac{1}{2}(PX + QX)$	$= 33° 0.4′.$

$$\therefore PX = \tfrac{1}{2}(PX + QX) + \tfrac{1}{2}(PX - QX) = 40° 1.6′.$$

Therefore the latitude of $X = 90° - 40° 1.6′ = 49° 58.4′$ N.

4. Denote the arcs SP, SZ, and PZ (see Fig., p. 135) by z, p, and s, and their sum by $2s'$.

Then in the triangle SPZ,

$$z = 90^\circ - 18^\circ 36' = 71^\circ 24', \quad p = 90^\circ - 14^\circ 18' = 75^\circ 42',$$
$$s = 90^\circ - 50^\circ 13' = 39^\circ 47'.$$

By Art. 158,

$$\cos \tfrac{1}{2} P = \sqrt{\frac{\sin s' \sin (s' - p)}{\sin s \sin z}}.$$

From the data,

$$s' = 93^\circ 26.5', \quad s' - p = 17^\circ 44.5'.$$

$$\begin{aligned} \log \sin s' &= 9.9992 \\ \log \sin (s' - p) &= 9.4839 \\ \log \csc s &= 0.1939 \\ \log \csc z &= 0.0233 \\ & \overline{2)9.7003} \\ \log \cos \tfrac{1}{2} P &= 9.8501 \end{aligned}$$

$$\therefore \tfrac{1}{2} P = 44^\circ 55', \text{ and } P = 89^\circ 50'.$$

To $89^\circ 50'$ corresponds 5 h. 59 m. 20 s. of time.

Therefore the hour of the day is 6 h. 0 m. 40 s. A.M.

The difference between the local and Greenwich time is 2 h. 59 m. 20 s., which corresponds to $44^\circ 50'$ of longitude.

Hence the longitude of the vessel is $44^\circ 50'$ W.

5. Denote the arcs SP, SZ, and PZ (see Fig., p. 135) by z, p, and s.

Then in the triangle SPZ,

$$z = 90^\circ + 12^\circ = 102^\circ, \quad s = 90^\circ - 37^\circ 48' = 52^\circ 12', \quad \angle P = 60^\circ.$$

By Art 162, we have:

$$\tan \tfrac{1}{2} (Z - S) = \frac{\sin \tfrac{1}{2} (z - s)}{\sin \tfrac{1}{2} (z + s)} \cot \tfrac{1}{2} P,$$

$$\tan \tfrac{1}{2} (Z + S) = \frac{\cos \tfrac{1}{2} (z - s)}{\cos \tfrac{1}{2} (z + s)} \cot \tfrac{1}{2} P.$$

From the data, $\tfrac{1}{2} (z - s) = 24^\circ 54'$, $\tfrac{1}{2} (z + s) = 77^\circ 6'$, $\tfrac{1}{2} P = 30^\circ$.

$$\begin{aligned} \log \sin \tfrac{1}{2} (z - s) &= 9.6243 \\ \log \csc \tfrac{1}{2} (z + s) &= 0.0111 \\ \log \cot \tfrac{1}{2} P &= 0.2386 \\ \log \tan \tfrac{1}{2} (Z - S) &= 9.8740 \end{aligned}$$

$$\therefore \tfrac{1}{2} (Z - S) = 36^\circ 48.1'.$$

$$\begin{aligned} \log \cos \tfrac{1}{2} (z - s) &= 9.9577 \\ \log \sec \tfrac{1}{2} (z + s) &= 0.6512 \\ \log \cot \tfrac{1}{2} P &= 0.2386 \\ \log \tan \tfrac{1}{2} (Z + S) &= 0.8475 \end{aligned}$$

$$\therefore \tfrac{1}{2} (Z + S) = 81^\circ 54.8'.$$

$$\tan \tfrac{1}{2}p = \frac{\sin \tfrac{1}{2}(Z+S)}{\sin \tfrac{1}{2}(Z-S)} \tan \tfrac{1}{2}(z-s).$$

$\log \sin \tfrac{1}{2}(Z+S) = 9.9957$
$\log \csc \tfrac{1}{2}(Z-S) = 0.2225$
$\log \tan \tfrac{1}{2}(z-s) = 9.6667$
$\log \tan \tfrac{1}{2}p = 9.8849$
$\therefore \tfrac{1}{2}p = 37^\circ 29.6'$, and $p = 74^\circ 59.2'$.

Therefore the altitude of the sun is $90^\circ - 74^\circ 59.2'$, or $15^\circ 0.8'$.

6. Denote the arcs SP, SZ, and PZ (see Fig., p. 135) by z, p, and s, and their sum by $2s'$.

Then in the triangle SPZ,
$z = 90^\circ + 3^\circ = 93^\circ$, $p = 90^\circ - 25^\circ 46' = 64^\circ 14'$, $s = 90^\circ + 37^\circ 49' = 127^\circ 49'$.

By Art. 158, $$\cos \tfrac{1}{2}P = \sqrt{\frac{\sin s' \sin (s'-p)}{\sin s \sin z}}.$$

From the data, $s' = 142^\circ 31.5'$, $s' - p = 78^\circ 17.5'$.

$\log \sin s' = 9.7842$
$\log \sin (s'-p) = 9.9908$
$\log \csc s = 0.1024$
$\log \csc z = 0.0006$
$2)\overline{9.8780}$
$\log \cos \tfrac{1}{2}P = 9.9390$
$\therefore \tfrac{1}{2}P = 29^\circ 40'$, and $P = 59^\circ 20'$.

To $59^\circ 20'$ corresponds 3 h. 57 m. 20 s. of time.
Therefore the hour of the day is 8 h. 2 m. 40 s. A.M.

7. Denote the arcs SP, SZ, and PZ (see Fig., p. 135) by z, p, and s.

Then in the triangle SPZ,
$z = 90^\circ - 15^\circ = 75^\circ$, $p = 90^\circ$, $s = 90^\circ - 42^\circ 21' = 47^\circ 39'$.

Therefore in the polar triangle of SPZ, we have
$Z = 180^\circ - 75^\circ = 105^\circ$, $P' = 180^\circ - 90^\circ = 90^\circ$,
$S' = 180^\circ - 47^\circ 39' = 132^\circ 21'$.

By Art. 144, $$\underset{+}{\cos p'} = \underset{-}{\cot S'} \underset{-}{\cot Z'}.$$

$\log \cot S' = 9.9598$
$\log \cot Z' = 9.4281$
$\log \cos p' = 9.3879$
$\therefore p' = 75^\circ 51.6'$.

Then in the triangle SPZ, we have
$P = 180^\circ - p' = 104^\circ 8.4'$.

To $104^\circ 8.4$ corresponds 6 h. 56 m. 33.6 s. of time.
Therefore the hour of the day is 5 h. 3 m. 26.4 s. A.M.

USE OF THE TABLES.

Page 2.

3. $\log 80 = 1.9031$.

4. $\log 6.3 = 0.7993$.

5. $\log 298 = 2.4742$.

6. $\log .902 = 9.9552 - 10$.

7. Mantissa of $772 = .8876$
   $.3 \times 6 = 2$
   $\therefore \log .7723 = 9.8878 - 10$

8. Mantissa of $105 = .0212$
   $.6 \times 41 = 25$
   $\therefore \log 1056 = 3.0237$

9. Mantissa of $329 = .5172$
   $.4 \times 13 = 5$
   $\therefore \log 3.294 = 0.5177$

10. Mantissa of $520 = .7160$
    $.5 \times 8 = 4$
    $\therefore \log .05205 = 8.7164 - 10$

11. Mantissa of $200 = .3010$
    $.8 \times 22 = 18$
    $\therefore \log 20.08 = 1.3028$

12. Mantissa of $924 = .9657$
    $.61 \times 4 = 2$
    $\therefore \log 92461 = 4.9659$

13. Mantissa of $403 = .6053$
    $.22 \times 11 = 2$
    $\therefore \log .40322 = 9.6055 - 10$

14. Mantissa of $717 = .8555$
    $.8 \times 6 = 5$
    $\therefore \log .007178 = 7.8560 - 10$

15. Mantissa of $518 = .7143$
    $.09 \times 9 = 1$
    $\therefore \log 5.1809 = 0.7144$

16. Mantissa of $103 = .0128$
    $.65 \times 42 = 27$
    $\therefore \log 1036.5 = 3.0155$

17. Mantissa of $866 = .9375$
    $.76 \times 5 = 4$
    $\therefore \log .086676 = 8.9379 - 10$

18. Mantissa of $115 = .0607$
    $.07 \times 38 = 3$
    $\therefore \log .11507 = 9.0610 - 10$

Page 4.

4. Number corresponding to $1.8055 = 63.9$.

5. Number corresponding to $9.4487 - 10 = .281$.

6. 0.2165
.2148 = mantissa of 164
$\frac{17}{27}$ = 6
∴ Number corresponding = 1.646

7. 3.9487
.9484 = mantissa of 888
$\frac{3}{5}$ = 6
∴ Number corresponding = 8886

8. Number corresponding to
2.7364 = 545.

9. 8.1648 − 10
.1644 = mantissa of 146
$\frac{4}{29}$ = 1
∴ Number corresponding = .01461

10. 7.5209 − 10
.5198 = mantissa of 331
$\frac{11}{13}$ = 8
∴ Number corresponding = .003318

11. 4.0095
.0086 = mantissa of 102
$\frac{9}{42}$ = 21
∴ Number corresponding = 10221

12. 0.9774
.9773 = mantissa of 949
$\frac{1}{4}$ = 2
∴ Number corresponding = 9.492

13. 9.3178 − 10
.3160 = mantissa of 207
$\frac{18}{21}$ = 9
∴ Number corresponding = .2079

14. 1.6482
.6474 = mantissa of 444
$\frac{8}{10}$ = 8
∴ Number corresponding = 44.48

15. 7.0450 − 10
.0414 = mantissa of 110
$\frac{36}{39}$ = 9
∴ Number corresponding = .001109

16. 4.8016
.8014 = mantissa of 633
$\frac{2}{7}$ = 29
∴ Number corresponding = 63329

17. 8.1144 − 10
.1139 = mantissa of 130
$\frac{5}{34}$ = 1
∴ Number corresponding = 0.1301

18. 2.7015
.7007 = mantissa of 502
$\frac{8}{9}$ = 9
∴ Number corresponding = 502.9

Page 6.

3. $\log \tan 35^\circ 10' = 9.8479$
$2.7 \times 9 = \underline{\quad 24}$
$\therefore \log \tan 35^\circ 19' = 9.8503$

4. $\log \sin 61^\circ 50' = 9.9453$
$.6 \times 8 = \underline{\quad 5}$
$\therefore \log \sin 61^\circ 58' = 9.9458$

5. $\log \cot 12^\circ 30' = 0.6542$
$5.9 \times 4 = \underline{\quad 24}$
$\therefore \log \cot 12^\circ 34' = 0.6518$

6. $\log \cos 26^\circ 50' = 9.9505$
$.6 \times 6 = \underline{\quad 4}$
$\therefore \log \cos 26^\circ 56' = 9.9501$

7. $\log \tan 82^\circ 0' = 0.8522$
$9.3 \times 3\frac{1}{3} = \underline{\quad 31}$
$\therefore \log \tan 82^\circ 3' 20'' = 0.8553$

8. $\log \sin 55^\circ 10' = 9.9142$
$.9 \times 1.8 = \underline{\quad 2}$
$\therefore \log \sin 55^\circ 11.8' = 9.9144$

9. $\log \cos 30^\circ 40' = 9.9346$
$.8 \times 2.5 = \underline{\quad 2}$
$\therefore \log \cos 30^\circ 42.5' = 9.9344$

10. $\log \cot 48^\circ 0' = 9.9544$
$2.5 \times 3\frac{43}{60} = \underline{\quad 9}$
$\therefore \log \cot 48^\circ 3' 43'' = 9.9535$

Page 7.

3. 0.9164
$0.9109 = \log \tan \ 83^\circ 0'$
$\frac{55}{10.5} = \underline{\quad 5.2}$
$\therefore$ Angle corresponding $= 83^\circ 5.2'$

4. 9.9238
$9.9236 = \log \cos 33^\circ \ 0'$
$\frac{2}{.8} = \underline{\quad 2.5}$
$\therefore$ Angle corresponding $= 32^\circ 57.5'$

5. 9.8630
$9.8629 = \log \sin \ 46^\circ 50'$
$\frac{1}{1.2} = \underline{\quad 0.8}$
$\therefore$ Angle corresponding $= 46^\circ 50.8'$

6. 0.2154
$0.2127 = \log \cot 31^\circ 30'$
$\frac{27}{2.8} = \underline{\quad 9.6}$
$\therefore$ Angle corresponding $= 31^\circ 20.4'$

7. 9.2279
$9.2251 = \log \cos 80^\circ 20'$
$\frac{28}{7.3} = \underline{\quad 3.8}$
$\therefore$ Angle corresponding $= 80^\circ 16.2'$

8. 9.4700
$9.4669 = \log \cot 73^\circ 40'$
$\frac{31}{4.7} = \underline{\quad 6.6}$
$\therefore$ Angle corresponding $= 73^\circ 33.4'$

9. 9.1891
$9.1863 = \log \sin \ 8^\circ 50'$
$\frac{28}{8} = \underline{\quad 3.5}$
$\therefore$ Angle corresponding $= \ 8^\circ 53.5'$

10. 0.0502
$0.0481 = \log \tan 48^\circ 10'$
$\frac{21}{2.5} = \underline{\quad 8.4}$
$\therefore$ Angle corresponding $= 48^\circ 18.4'$

Page 8.

2. $\log \sin 65° 10' = 9.9579$
$.5 \times 2 = \quad 1$
$\log \sin 65° 12' = 9.9580$
$\therefore \log \csc 65° 12' = 0.0420$

3. $\log \cos 80° 0' = 9.2397$
$7.3 \times 7.3 = \quad 53$
$\log \cos 80° 7.3' = 9.2344$
$\therefore \log \sec 80° 7.3' = 0.7656$

4. $9.5997 = \log \cos.$
$9.5978 = \log \cos 66° 40'$
$\frac{19}{2.9} = \quad 6.6$
$\therefore$ Angle corresponding $= 66° 33.4'$

5. $9.8112 = \log \sin.$
$9.8111 = \log \sin 40° 20'$
$\frac{1}{1.4} = \quad 0.7$
$\therefore$ Angle corresponding $= 40° 20.7'$

Page 10.

3. nat. $\sin 3° 30' = .06105$
$.00029 \times 2 = .00058$
nat. $\sin 3° 32' = .06163$
mantissa of $616 = .7896$
$7 \times .3 = \quad 2$
$\therefore \log \sin 3° 32' = 8.7898$

4. nat. $\cos 88° 10' = .03199$
$.000291 \times 7 = .00204$
nat. $\cos 88° 17' = .02995$
mantissa of $299 = .4757$
$14 \times .5 = \quad 7$
$\therefore \log \cos 88° 17' = 8.4764$

5. nat. $\tan 2° 20' = .04075$
$.000291 \times 8.2 = .00239$
nat. $\tan 2° 28.2' = .04314$
mantissa of $431 = .6345$
$10 \times .4 = \quad 4$
$\therefore \log \tan 2° 28.2' = 8.6349$

6. nat. $\tan 50' = .014545$
$.000291 \times 5\frac{14}{60} = .001523$
nat. $\tan 55' 14'' = .016068$
mantissa of $160 = .2041$
$27 \times .68 = \quad 18$
$\log \tan 55' 14'' = 8.2059$
$\therefore \log \cot 55' 14'' = 1.7941$

7. 7.8702
$.8698 =$ mantissa of 741
$\frac{4}{6} = \quad 7$
Number corresponding $= .007417$

$.007417 =$ nat. sin.
$.005818 =$ nat. sin $20'$
$\frac{.001599}{.0002909} = 5.497' = \quad 5' 29.8''$
$\therefore$ Angle corresponding $= 25' 29.8''$

8. 8.6150
$.6149 =$ mantissa of 412
$\frac{1}{11} = \quad 1$
Number corresponding $= .04121$

$.04121 =$ nat. cos.
$.04071 =$ nat. cos $87° 40'$
$\frac{.00050}{.000291} = 1.718' = \quad 1' 43.1''$
$\therefore$ Angle corresp. $= 87° 38' 16.9''$

9. 8.2892
.2878 = mantissa of 194

$\frac{14}{22}$ = 64

Number corresponding = .019464

.019464 = nat. cot.
.017455 = nat. cot 89° 0′

$\frac{.002009}{.0002905}$ = 6.916′ = 6′ 55″

∴ Angle corresp. = 89° 53′ 5″

10. 8.2131 = cologarithm.
.2122 = mantissa of 163

$\frac{9}{26}$ = 35

Number corresponding = .016335

.016335
.014545 = nat. tan 50′

$\frac{.001790}{.000291}$ = 6.151′ = 6′ 9.1″

56′ 9.1″

∴ Angle corresp. = 89° 3′ 50.9″

Page 11.

3. nat. sin 17° 30′ = .3007
.00028 × 3 = .0008

∴ nat. sin 17° 33′ = .3015

4. nat. cos 75° 40′ = .2476
.00029 × 8.3 = .0024

∴ nat. cos 75° 48.3′ = .2452

5. .7385
.7373 = nat. sin 47° 30′

$\frac{.0012}{.0019}$ = 6.3

∴ Angle corresponding = 47° 36.3′

6. .9280
.9272 = nat. cos 22° 0′

$\frac{.0008}{.0011}$ = 7.3

∴ Angle corresponding = 21° 52.7′